MACHAKOS UNIVERSITY

University Examinations 2018/2019

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS STATISTICS AND ACTUARIAL SCIENCE

FIRST YEAR SECOND SEMESTER EXAMINATION FOR

DIPLOMA IN EDUCATION

SMA 0104: LINEAR ALGEBRA 1

DATE: 10/5/2019 **TIME: 8:30 – 10:30 AM**

INSTRUCTIONS:

Answer Question ONE and any other TWO questions

QUESTION ONE (COMPULSORY) (30 MARKS)

a) Given the matrices

$$A = \begin{bmatrix} 2 & 12 \\ -4 & 5 \end{bmatrix} \qquad B = \begin{bmatrix} 2 & -5 \\ 0 & 9 \end{bmatrix}$$

Determine:

i) 4A+3B

ii) $(AB)^{-1}$ (10 marks)

b) Solve the following simultaneous equation using the inverse matrix method

$$12x + 14y = 126$$

$$15x + 10y = 90$$

(6 marks)

c) Define the following

i) skew symmetric

ii) square matrix

iii) Transpose of a matrix

iv) diagonal matrix (4 marks)

d) Given matrix

$$A = \begin{bmatrix} 1 & 2 & 0 \\ 2 & 1 & 1 \\ 1 & 2 & 4 \end{bmatrix} \quad and \quad B = \begin{bmatrix} -1 & 1 & 3 \\ 2 & 2 & 1 \\ 1 & 1 & 2 \end{bmatrix}$$

i) Find $AB - A$ (4 marks)

ii) $(BA)^{-1}$ (6 marks)

QUESTION TWO (20 MARKS)

a) Given the matrices

$$A = \begin{bmatrix} 3 & -2 \\ 4 & 0 \end{bmatrix} \qquad B = \begin{bmatrix} 5 & 7 \\ 2 & 3 \end{bmatrix} \qquad C = \begin{bmatrix} 3 & 2 \\ 2 & 3 \end{bmatrix}$$

i) Show that $A(B + C) = AB + AC$

ii) Determine $B^{-1}C$ (14 marks)

b) The matrix $X = \begin{bmatrix} a \\ b \\ \end{bmatrix}$ satisfies the relationship $ABX = C$ where

$$A = \begin{bmatrix} -2 & 3 \\ 3 & -5 \end{bmatrix} \qquad B = \begin{bmatrix} 4 & 5 \\ 3 & 2 \end{bmatrix} \qquad C = \begin{bmatrix} 3 \\ 18 \end{bmatrix}$$

Evaluate a and b

(6 marks)

QUESTION THREE (20 MARKS)

a) Given that

$$A = \begin{bmatrix} 2 & 0 & 1 \\ 3 & 1 & 2 \\ 1 & 2 & 1 \end{bmatrix} \qquad B = \begin{bmatrix} 0 & 2 & 1 \\ 2 & 2 & 1 \\ 3 & 1 & 1 \end{bmatrix} \quad and \quad C = \begin{bmatrix} 2 & 4 & 2 \\ 6 & 9 & 7 \\ 5 & 7 & 4 \end{bmatrix}$$

Determine

i) $M = AB - C$

ii) M^{-1}

iii) Use the results above to solve the simultaneous equations

$$x + y + z = 9$$
$$2x + y + z = 7$$
$$2x + 2 = 5$$

(15 marks)

b) Evaluate the determinant of the matrix

$$A = \begin{bmatrix} -3 & 1 & -1 \\ 1 & -5 & 1 \\ -1 & 1 & -3 \end{bmatrix}$$

(5 marks)

QUESTION FOUR (20 MARKS)

a) Three forces $F_1 \quad F_2 \quad and \quad F_3$ acting on a mechanical system satisfies the simultaneous equations

$$F_1 + 2F_2 + 3F_3 = 14$$
$$2F_1 + 3F_2 + F_3 = 11$$
$$F_1 + F_2 + 2F_3 = 9$$

Use the inverse of a matrix method to determine the magnitude of the three forces

(10 marks)

b) Determine the eigen values and eigen vectors for the equation $\boldsymbol{A}.\boldsymbol{x} = \lambda x$ where

$$A = \begin{bmatrix} 2 & 0 & 1 \\ -1 & 4 & -1 \\ -1 & 2 & 0 \end{bmatrix}$$

(10 marks)

QUESTION FIVE (20 MARKS)

a) Use the Gaussian elimination method to solve the set of linear equations

$$x - 4y - 2z = 21$$
$$2x + x + 2y = 3$$
$$3x + 2x - z = -2$$

(10 marks)

b) Given that $p = \begin{bmatrix} a & 2a \\ a-1 & a+1 \end{bmatrix}$ is a singular matrix determine the possible value of a

(4 marks)

c) Determine the Eigen values and Eigen vectors corresponding to the matrix

$$A = \begin{bmatrix} 1 & 2 & 2 \\ 1 & 3 & 1 \\ 2 & 2 & 1 \end{bmatrix}$$

(6 marks)