



# MACHAKOS UNIVERSITY

University Examinations 2020/2021 Academic Year

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

FIRST YEAR SPECIAL /SUPPLEMENTARY EXAMINATION FOR  
BACHELOR OF SCIENCE (ELECTRICAL AND ELECTRONICS ENGINEERING)

BACHELOR OF SCIENCE IN (MECHANICAL ENGINEERING)

BACHELOR OF SCIENCE IN (CIVIL ENGINEERING)

ECU 104: ENGINEERING MATHEMATICS 1

DATE: 26/3/2021

TIME: 2.00-4.00 PM

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## INSTRUCTIONS

Answer question ONE and any other TWO questions

Programmable calculators are prohibited

### QUESTION ONE (30 MARKS)

- a) Find the equation of a circle which has a diameter joining points (5,-1) and (-3,7) (4 marks)
- b) i Differentiate between a vector quantity and a scalar quantity (2 marks)  
ii Calculate the cross product between  $\vec{a} = (3, -3, 1)$  and  $\vec{b} = (4, 9, 2)$  (3 marks)
- c) Find the length of the tangent drawn from the point (2,5) to the circle  $x^2 + y^2 - 2x - 3y = 1$  (4 marks)
- d) Prove that the coordinates of the circle  $x^2 + y^2 - 16x - 12y + 75 = 0$  and  $5x^2 + 5y^2 - 32x - 24y + 75 = 0$  touch each other and find the equation of the common tangent at their point of contact. (5 marks)
- e) Find the length of the tangent drawn from the point (2,5) to the circle  $x^2 + y^2 - 2x - 3y = 1$  (5 marks)
- f) Derive the equation of the ellipse having its centre at the origin, one focus at (0,3) and the length of the semi major axis 5. (4 marks)

- g) Find the distance of the point  $(h, k)$  from the line  $x \cos \alpha + y \sin \alpha = p$  (3 marks)

### QUESTION TWO (20 MARKS)

- a) Show that triangle ABC, is isosceles where A, B and C are the points  $(3,7)$ ,  $(-4,1)$  and  $(-3,-2)$  respectively. (6 marks)
- b) Find the polar equation of a circle whose Cartesian equation is  $x^2 + y^2 = 4x$  (5 marks)
- c) Given the equation of a parabola as  $3x^2 - 9x - 5y - 2 = 0$ . Find
- i. The vertex (4 marks)
  - ii. Focus (2 marks)
  - iii. Equation of directrix (2 marks)
  - iv. Length of the latus vectum (1 mark)

### QUESTION THREE (20 MARKS)

- a) Find the sum of the following vectors
- i.  $(4,1)$  and  $(6,-7)$  (3 marks)
  - ii.  $(3,-1,-2)$  and  $(-2,3,-5)$  (3 marks)
- b) A person has to travel some distance. First he moves 30 km towards West from his initial point and then he takes a turn towards South and travels around 40 km. Find the magnitude and direction of the resultant vector. (6 marks)
- c) Calculate the dot product of  $\vec{a} = (1,2,3)$  and  $\vec{b} = (4, -5,6)$ . Do the vectors form an acute angle, right angle or obtuse angle? (5 marks)
- d) Find the magnitude of the vector  $\mathbf{a} = \mathbf{i} + 4\mathbf{j} + 8\mathbf{k}$  (3 marks)

### QUESTION FOUR (20 MARKS)

- a) P is the point  $(at_1^2, 2at_1)$  and Q the point  $(at_2^2, 2at_2)$  on a parabola  $y^2 = 4ax$ . The tangents at P and Q intersect at R. Show that the area of the triangle PQR is  $\frac{1}{2}a^2(t_1 - t_2)^3$  (10 marks)
- b) Given the ellipse whose equation is  $4x^2 + 9y^2 - 48x + 72y + 144 = 0$ , find its centre, semi-axes, vertices foci and equations of diretrices. (10 marks)

### QUESTION FIVE (20 MARKS)

- a) Find the coordinates of the vertices and foci, the equations of the diretrices, the equations of the asymptotes, the length of the latus vectum, the eccentricity and plot the graph of the hyperbola  $9x^2 - 16y^2 = 144$  (10 marks)

- b) Find the equations of the tangent and normal to the curve  $y = 2x^2 - 4x + 5$ , at the point where  $x = 2$  (5 marks)
- c) Show that  $A(4,1)$  and  $B(2,-3)$  are equidistant from the line  $2x + 5y = 1$  (5 marks)