



MACHAKOS UNIVERSITY

University Examinations for 2021/2022 Academic Year

DIRECTORATE OF TVET

THIRD YEAR FIRST TERM EXAMINATION FOR

DIPLOMA IN AUTOMOTIVE AND MECHANICAL ENGINEERING

ENGINEERING MATHEMATICS III

DATE: 25/7/2022

TIME: 8:30 – 11:30 AM

INSTRUCTIONS

Answer **ALL** questions

Each question carries **20** marks

QUESTION ONE (20 MARKS)

- a) Determine the full-range Fourier series expansion $f(t)$ of period 2π defined by

$$f(t) = \begin{cases} \frac{2}{3}t & 0 \leq t < \frac{1}{3}\pi \\ \frac{1}{3}(\pi - t) & \frac{1}{3}\pi \leq t < \pi \\ f(t + 2\pi) & \end{cases}$$

To what value does the series converge at $t = \frac{\pi}{3}$ (10 marks)

- b) A function $f(x)$ is defined by the data in the table below

X	1	2	3	4	5	6
F(x)	15	99	273	675	1403	2595

Use the Newton-Gregory difference formula to estimate $f(1.8)$ (10 marks)

QUESTION TWO (20 MARKS)

- a) If x_n is an approximation root of the equation $x^4 - 5x^2 + 7x - 8 = 0$

- i) Show using Newton-Raphson method that a better approximation x_{n+1} is given by

$$x_{n+1} = \frac{3x_n^4 - 5x_n^2 + 8}{4x_n^3 - 10x_n + 7}$$

- ii) Starting with $x_0 = 1.5$ find the root of the equation correct to four decimal places. (10 marks)

b) A function $f(x)$ is defined by the data in the table

x	1	2	3	4	5	6	7
F(x)	4	47	237	755	1853	3855	7157

Use Newton-Gregory difference formula to estimate $f(1.8)$ (10 marks)

QUESTION THREE (20 MARKS)

a) A function $f(x)$ is defined by

$$f(x) = \begin{cases} 2 - x & 0 < x < 4 \\ x - 6 & 4 < x < 8 \\ f(x + 8) \end{cases}$$

- i. Sketch the function in the range $-4 < x < 8$
- ii. Obtain the Fourier series for the function (14 marks)
- b) Expand a function $f(x) = x_1$ $0 < x < 3$ in a half range Sine series (6 marks)

QUESTION FOUR (20 MARKS)

a) Given the matrix

$$M = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 6 & 7 \\ 5 & 8 & 9 \end{pmatrix}$$

- i) Determine M^{-1} .
- ii) Hence solve the equations
- $$\begin{aligned} x + 2y + 3z &= -2 \\ 4x + 6y + 7z &= -7 \\ 5x + 8y + 9z &= -8 \end{aligned}$$
- (10 marks)

b) Use the method of determinants to solve

$$\begin{aligned} 5x - 2y + z &= -7 \\ 3x - y + 2z &= -3 \\ 3y - x + z &= 0 \end{aligned}$$

(10 marks)

QUESTION FIVE (20 MARKS)

a) Given $\mathbf{A} = x^2y^3\mathbf{i} - 2xyz^2\mathbf{j} + x^2z\mathbf{k}$

$\mathbf{B} = xy^2z\mathbf{i} + 3yz^2\mathbf{j} - xyz^2\mathbf{k}$ and that

$$\phi = xy^2z^3 - 3xy^2 + xyz^2$$

Determine at the point (1, -2, 1)

i. $\nabla\phi$

ii. $\nabla \cdot \mathbf{A}$

iii. $\nabla \times \mathbf{B}$

(10 marks)

b) If $\mathbf{OA} = 2\mathbf{i} + 3\mathbf{j} - \mathbf{k}$ and $\mathbf{OB} = \mathbf{i} - 2\mathbf{j} + 5\mathbf{k}$

Determine

i. The value of $\mathbf{OA} \cdot \mathbf{OB}$

(2 marks)

ii. The product $\mathbf{OA} \times \mathbf{OB}$ in terms of unit vectors

(2 marks)

iii. The cosine of the angle between $\mathbf{OA}$ and $\mathbf{OB}$

(6 marks)