MACHAKOS UNIVERSITY

University Examinations 2018/2019

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS, STATISTICS AND ACTUARIAL SCIENCE

FOURTH YEAR SUPPLEMENTARY EXAMINATION FOR

BACHELOR OF SCIENCE (MATHEMATICS AND COMPUTER SCIENCE)

BACHELOR OF SCIENCE (MATHEMATICS)

SMA 430: NUMERICAL ANALYSIS II

DATE: 26/9/2019 **TIME: 2:00 – 4:00 PM**

INSTRUCTIONS TO CANDIDATES

Attempt question ***one (compulsory)*** and any other ***two questions***.

QUESTION ONE (30 MARKS)

a) Solve the system of equations below using the inverse of the coefficients matrix method (5 marks)

$$x_1 + 2x_2 - x_3 = 2$$
$$3x_1 + 6x_2 + x_3 = 1$$
$$3x_1 + 3x_2 + 2x_3 = 3$$

b) Determine the Eigen values and the corresponding Eigen vectors of the following system. (5 marks)

$$10x_1 + 2x_2 + x_3 = \lambda x_1$$
$$2x_1 + 10x_2 + x_3 = \lambda x_2$$
$$2x_1 + x_2 + 10x_3 = \lambda x_3$$

c) Solve the system of equations below using Doolittle's method. (5 marks)

$$2x + 3y + z = 9$$
$$x + 2y + 3z = 6$$
$$3x + y + 2z = 8$$

d) Solve the system of equations

$$\begin{bmatrix} 2 & 1 & 1 & -2 \\ 4 & 0 & 2 & 1 \\ 3 & 2 & 2 & 0 \\ 1 & 3 & 2 & -1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} -10 \\ 8 \\ 7 \\ -5 \end{bmatrix}$$

Using the Gauss- Elimination method with partial pivoting (5 marks)

e) Consider the following matrix

$$A = \begin{bmatrix} 2 & 1 & 0 \\ 2 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

i.) Show that the matrix satisfies its own characteristics equation (5 marks)

ii.) Determine A^{-1} (5 marks)

QUESTION TWO (20 MARKS)

Consider the differential equation $\frac{dy}{dx} = 2e^x y$, $y(0) = 2$. Calculate $y(0.2)$ using Adams predictor corrector formula by calculating $y(0.1)$, $y(0.2)$ *and* $y(0.3)$ using the Euler's modified formula.

QUESTION THREE (20 MARKS)

Consider the approximate value of $I = \int_{-1}^{1} e^{-x^2} \cos x dx$ obtain the value using.

a) Gauss-Legendre integration method for $n = 2$

(10 marks)

b) Radau integration method for $n = 2$

(10 marks)

QUESTION FOUR (20 MARKS)

Consider the solution of the system of equations

$$\begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 7 \\ 1 \\ 1 \end{bmatrix}$$

i.) Set up SOR scheme for the system (4 marks)

ii.) Determine the optimal relaxation factor and the rate of convergence(5 marks)

iii.) Perform three iterations of the SOR scheme given by

$$X^{(k+1)} = Hx^{(k)} + C, \quad k = 0,1,2,....$$ (5 marks)

iv.) Perform three iterations of the SOR scheme given by

$$(D + \omega L)V^{(k)} = \omega r^{(k)}$$

$$V^{(k)} = \omega (D + \omega L)^{-1} r^{(k)}$$ (6 marks)

Take the initial approximation as $x^0 = 0.$ the exact solution is

$x_1 = 6$, $x_2 = 5$, $x_3 = 3$

QUESTION FIVE (20 MARKS)

a) Consider the initial value problem (I .v. p) given by

$y' = 2x + 3y \quad ; \; y(0) = 1$

Use Taylors series second order method to get $y(0.4)$ with step size length $h = 0.1$

(10 marks)

b) Let $f(x) = \ln x$ and $x_0 = 1.8$ for $h > 0$,evaluate $f'(x)$

(5 marks)

c) Calculate $\int_0^{\frac{1}{2}} \frac{x}{\sin x} dx$ using Romberg integration with step size $h = \frac{1}{16}$ (5 marks)