MACHAKOS UNIVERSITY

University Examinations 2018/2019

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS, STATISTICS AND ACTUARIAL SCIENCE

SECOND YEAR SECOND SEMESTER EXAMINATION FOR

BACHELOR OF SCIENCE (ELECTRICAL AND ELECTRONICS ENGINEERING)

BACHELOR OF SCIENCE (MECHANICAL ENGINEERING)

BACHELOR OF SCIENCE (CIVIL ENGINEERING)

ECU 203: ENGINEERING MATHEMATICS VIII

DATE: 15/5/2019 **TIME: 8:30 – 10:30 AM**

INSTRUCTIONS:

Attempt question ***one (compulsory)*** and any other ***two questions***.

QUESTION ONE (COMPULSORY) (30 MARKS)

a) Calculate $P_4(x)$ using the Rodrigues formula for the Legendre functions. (5 marks)

b) Determine the polynomial solution to the Legendre equation

$$(1-x^2)y''-2xy'+12y=0$$ (5 marks)

c) Evaluate $\int J_5(x)dx$ (5 marks)

d) Using the Bessel function identities, express $[J_2(x)]'$ in terms of $J_0(x)$ *and* $J_1(x)$. (5 marks)

e) Determine the Laplace transform for;

i) $f(t)=e^{at}$ (5 marks)

ii) $f(x)=\cosh 2t$ (5 marks)

QUESTION TWO (20 MARKS)

a) Use Laplace transforms method to solve the second order differential equation

$\frac{d^2x}{dt^2} - 3\frac{dx}{dt} + 2x = 2e^{3t}$ given that at $t = 0$; $x = 5$ and $\frac{dx}{dt} = 7$ (12 marks)

b) Determine the inverse of $\frac{5s+1}{(s-4)(s+3)}$ (8 marks)

QUESTION THREE (20 MARKS)

a) Use the Bessel's function identities to express $J_3(x)$ in terms of

$J_0(x)$ and $J_1(x)$ (5 marks)

b) Evaluate $\int x^3 J_0 dx$ (5 marks)

c) Determine $J_{\pm\frac{3}{2}}$ in terms of the elementary functions $\sin x$ and $\cos x$ (4 marks)

d) Determine the general solution to Bessel's equation of order zero. (6 marks)

QUESTION FOUR (20 MARKS)

a) Determine the Fourier's series expansion for;

$$f(x) = \begin{cases} 0 & ; -\pi \le x < \frac{-\pi}{2} \\ \cos x & ; \frac{-\pi}{2} \le x < \frac{\pi}{2} \\ 0 & ; \frac{\pi}{2} \le x < \pi \end{cases}$$ (4 marks)

b) Determine a Fourier cosine series for $f(x) = e^x$ on $(0, \pi)$ (4 marks)

c) i) Define a Dirac delta function (1 mark)

ii) State the convolution theorem of Fourier transforms. (3 marks)

iii) Given the functions $f(t)$ and $g(t)$ and their convolution is;

$$f(t) * g(t) = \int_{-\infty}^{\infty} f(x)g(t-x)dx = h(t)$$

Where $*$ denotes the convolution operation.

If $f(t) = u(t)$ and $g(t) = \begin{cases} \sec^2 t & |t| < \frac{\pi}{4} \\ 0 & otherwise \end{cases}$

Where $u(t)$ is the Heaviside function.

Show that $u(t) = \dfrac{1+\tan^2 t}{1+\tan t}$ (4 marks)

d) Evaluate the inverse transform of $f(\omega) = \dfrac{1}{2\pi(a+j\omega)^2}$ where $a>0$

(4 marks)

QUESTION FIVE (20 MARKS)

a) Consider the expression $f(t) = L^{-1}\left\{\dfrac{2}{s} + \dfrac{3e^{-s}}{s^2} - \dfrac{3e^{-3s}}{s^2}\right\}$

i) Determine $f(t)$ (6 marks)

ii) Sketch the graph of $f(t)$ (7 marks)

b) When $p = n$ an integer $\alpha_1 = \alpha_2 = n$; $J_n(x)$ *and* $J_{-n}(x)$ and are linearly dependent.

Show that $J_{-n}(x) = (-1)^n J_n(x)$ (7 marks)