MACHAKOS UNIVERSITY

University Examinations 2020/2021 Academic Year

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

SECOND YEAR SPECIAL /SUPPLEMENTARY EXAMINATION FOR

BACHELOR OF SCIENCE (MATHEMATICS)

SMA 230: VECTOR ANALYSIS

DATE: 26/3/2021 **TIME: 8.30-10.30 AM**

INSTRUCTIONS TO CANDIDATES

Answer question 1 and any other two questions

QUESTION ONE (30 MARKS)

a) Define a vector quantity and give four examples (2 marks)

b) Given $V=(x+2y+az)i+(bx-3y-z)j+(4x+cy+2z)k$ is an irrotational vector field, find a, b, and c (4 marks)

c) If two vectors $\vec{A}=2i+\sqrt{3}j-3k$, $B=6i-3j+2k$ are inclined to each other at angle θ. Find the value of θ (4 marks)

d) Find the area of the triangle with vertices $A(1,3,2)$, $B(2,-1,1)$, $C(-1,2,3)$ (4 marks)

e) Find the projection of the vector $7i+5j+2k$ on the line passing through the points $A(2,3,-1)$, $B(-3,-5,4)$ (4 marks)

f) Find the directional derivative of $\phi(x,y,z)=4xz^3-3x^2y^2z$ at $(1,1,1)$ in the direction $3i-2j+5k$ (4 marks)

g) Given the scalar function $\phi(x,y,z)=xy+y^2z$ and the curve C defined by $x=t^2$, $y=2t$, $z=t+5$ between points $A(0,0,5)$, $B(1,2,6)$

i. Transform the integral $\oint_C \phi d\vec{r}$ into an integral involving t only (3 marks)

ii. Determine the interval of parameter t from the given parametric equations (2 marks)

iii. Lastly, evaluate $\oint_C \phi d\vec{r}$ along the given curve (3 marks)

QUESTION TWO (20 MARKS)

a) Find the unit vector perpendicular to both $a+b$ and $b+c$ where $a = i + j - k$; $b = -i + 2j + k$; $c = i - 2j + k$ (4 marks)

b) At any point of the curve $x = 3\cos t$, $y = 3\sin t$, $z = 4t$, find:

i. Tangent Vector (1 mark)

ii. Unit Tangent vector (2 marks)

iii. Normal Vector (2 marks)

iv. Unit Normal vector (2 marks)

c) Show that $\vec{F} = (y^2 + 2xz^2)i + (2xy - z)j + (2x^2 z - y + 2z)k$ is irrotational and hence find its scalar potential (9 marks)

QUESTION THREE (20 MARKS)

a) The position vectors of the points A, B, C, D are $3i - 2j - k$, $2i + 3j - 4k$, $-i + j + 2k$, $4i + 5j + \lambda k$ respectively. If the four points lie in a plane find the value of λ (5 marks)

b) Use Green's theorem to evaluate

$$\int_C (x^2 + xy)dx + (x^2 + y^2)dy$$

Where c is a square formed by the lines $y = \pm 1$, $x = \pm 1$ (7 marks)

c) If $F = i\sin 2t + je^{3t} + k(t^3 - 4t)$, then when $t = 1$, find

i. $\dfrac{dF}{dt}$ (2 marks)

ii. $\dfrac{d^2F}{dt^2}$ (2 marks)

iii. $\left|\dfrac{dF}{dt}\right|$ (2 marks)

iv. $\left|\dfrac{d^2F}{dt^2}\right|$ (2 marks)

QUESTION FOUR (20 MARKS)

a) A particle is acted upon by a constant forces $4i+j-3k$ and $3i+j-k$ is displaced from the point $A \equiv i+2j+3k$ to the point $B \equiv 5i+4j+k$. Find the work done (4 marks)

b) A particle moves in a space curve $x=2t^2$, $y=t^2-4t$, $z=3t-5$. Determine its velocity and acceleration at time $t=1$ in the direction $i-j+2k$ (4 marks)

c) If $F=x^2yi+xzj-2yzk$, evaluate $\int_C F \bullet dr$ between $A(0,0,0)$ and $B(4,2,1)$ along the curve C having parametric equations $x=4t$, $y=2t$, $z=t^3$ (6 marks)

d) Apply Stoke's theorem to calculate

$$\int_C 4ydy+2zdy+6ydz$$

Where c is the curve of the intersection of $x^2+y^2+z^2=6z$ and $z=x+3$ (6 marks)

QUESTION FIVE (20 MARKS)

a) If $\vec{a}=2i-j+k$; $\vec{b}=3i+j-k$; $\vec{c}=i+\lambda j$, find the value of λ for which $\vec{a}\bullet\left(\vec{b}\times\vec{c}\right)=-25$

(2 marks)

b) A particle is displaced from the point whose position is $5i-5j-7k$ to the point whose position vector is $6i+2j-2k$ under the action of a number constant forces defined by $10i-j+11k$, $4i+5j+6k$, and $-2i+j-9k$. Show that the total work done by force is $87units$ (4 marks)

c) If $\phi(x,y,z)=xy^2z$ and $A(x,y,z)=xzi-xy^2j+yz^2k$, find $\dfrac{\partial^3(\phi A)}{\partial x^2 \partial z}$ at $(2,-1,1)$ (4 marks)

d) Use divergence theorem to evaluate

$$\int \vec{A}\bullet d\vec{S},$$

Where $A=x^3i+y^3j+z^3k$ and S is the surface of the sphere $x^2+y^2+z^2=a^2$ (10 marks)