MACHAKOS UNIVERSITY

University Examinations 2023/2024

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF PHYSICAL SCIENCES

THIRD YEAR FIRST SEMESTER EXAMINATION FOR

BACHELOR OF SCIENCE IN APPLIED PHYSICS AND TECHNOLOGY)

SPT 307: LINEAR AND NON-LINEAR OPTICS

DATE: **TIME:**

INSTRUCTIONS:

- The paper consists of **two** sections.
- Section **A** is **compulsory** (30 marks).
- Answer any **two** questions from section **B** (each 20 marks).

CONSTANTS

$\epsilon_0 = 8.8542 \times 10^{-12}$ F/m, the permittivity of free space

$\mu_0 = 4\pi \times 10^{-7}$H/m, the permeability of vacuum

QUESTION ONE (30 MARKS)

(a) Discuss the following optical phenomena

i) Four wave-mixing (3 marks)

ii) Stimulated Raman Scattering (3 marks)

(b) Describe three distinguishing characteristics of a hologram. (6 marks)

(c) Show that the Poynting vector $\boldsymbol{S}$ is given by (5 marks)

$$\boldsymbol{S} = \frac{1}{\mu_o} \vec{E} \times \vec{B}$$

(d) Suppose we have an electric field given by $\mathbf{E} = (\alpha x^2 y^3 \hat{\mathbf{x}} + \beta z^4 \hat{\mathbf{y}}) \cos \omega t$. Calculate

i. The charge density $\rho(x, y, z, t)$ using Gauss' law for electricity. (4 marks)

ii. The magnetic field $B(x, y, z, t)$ using Faraday's law. (4 marks)

(e) Separate the time-dependent Schrödinger equation $-\frac{\hbar^2}{2m}\frac{\partial^2\psi(x,t)}{\partial x^2} + V\psi(x,t) = i\hbar\frac{\partial}{\partial t}\psi(x,t)$ into two equations, one equation depending on *x* alone and the other on *t* alone. (5 marks)

QUESTION TWO (20 MARKS)

(a) Describe the following effects

i) Pockell's effect. (2 marks)

ii) Optical Kerr effect. (2 marks)

(b) State the Maxwell's equation in

i) Free space (2 marks)

ii) In a medium (2 marks)

(c) Derive the wave equation from the Maxwell's equations (6 marks)

(d) A time-varying magnetic field is given by

$\boldsymbol{B} = B_0 \cos \omega t \, \boldsymbol{a}_y$,

where B_0 is a constant.

Calculate the induced *e.m.f* around the rectangular loop C in the xz-plane bounded by the lines $x = 0, x = a, z = 0$ and $z = b$ as shown in Figure 1.

(6 marks)

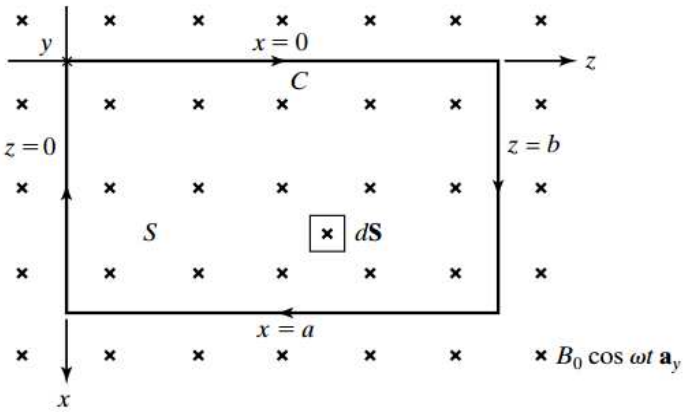

Figure 1

QUESTION THREE (20 MARKS)

(a) Define the following states of polarization of waves

i. Elliptical polarization (2 marks)

ii. Circular polarization (2 marks)

(b) Calculate the Jones matrix for a quarter-wave plate at $\theta = 45^0$, and determine its effect on horizontally polarized light. (4 mark)

(c) Consider electromagnetic waves in free space of the form

$\boldsymbol{E}(x, y, z, t) = E_0(x, y)e^{ikz - i\omega t}$,

$\boldsymbol{B}(x, y, z, t) = B_0(x, y)e^{ikz - i\omega t}$,

where E_0 and B_0 are in the xy plane.

Derive the relation between k and ω as well as the relation between $E_0(x, y)$ and $B_0(x, y)$.

(6 marks)

(d) Write down the time-independent Schrödinger equation for a particle of mass m in a one-dimensional harmonic oscillator potential centered at x = 0. Show that the spatial wave function

$$\psi(x) = A\, exp\left(-\frac{1}{2}\,\alpha\, x^2\right)$$

is an energy eigenfunction with total energy eigenvalue $E = \frac{1}{2}\hbar\omega$, where ω is the angular frequency of the corresponding classical oscillator. (6 marks)

QUESTION FOUR (20 MARKS)

a) Define the following terms

i. Dispersion (2 marks)

ii. Phase Matching (2 marks)

b) Solve the Maxwell equation for the electric field of a uniform sinusoidal plane wave with radian frequency ω propagating along the Z-axis in the medium with $\sigma = 0$. Determine the following physical quantities:

i. Propagation constant, (3 marks)

ii. Phase velocity, Group velocity, Wave impedance, and (3 marks)

iii. Poynting vector. (3 marks)

c) The cutoff frequency of the TE_{10} mode is 10 GHz, whereas the cutoff frequency of the TE_{03} mode is 60 GHz. The waveguide is filled with glass dielectric of $\varepsilon = 4\varepsilon_0$.

i. Calculate the dimensions a and b and the cutoff frequency of the TE_{10} mode in air. (3 marks)

ii. For an air filled waveguide of the same dimensions as calculated in part (i), calculate the number of possible modes if the excitation frequency is 65 GHz. (4 marks)

QUESTION FIVE (20 MARKS)

a) Two optical beams E_1 and E_2 with wavelengths of 1.0 and 0.6 μm respectively are incident on a nonlinear material as shown in Figure 2.

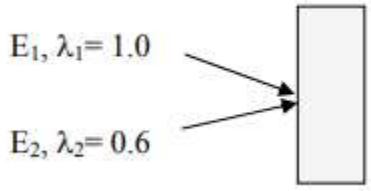

Figure 2

Assuming a $\chi^{(2)}$ nonlinearity, calculate the new wavelengths that can possibly be generated in this Material. (8 marks)

b) The above nonlinear material is now replaced with a centro-symmetric material calculate the dominant nonlinear susceptibility. (4 marks)

c) Write down the differential forms of Maxwell's equations in a free space and explain what they mean. (8 marks)