MACHAKOS UNIVERSITY

University Examinations for 2024/2025 Academic Year

SCHOOL OF PURE AND APPLIED SCIENCES
DEPARTMENT OF MATHEMATICS, STATISTICS AND ACTUARIAL SCIENCES

THIRD YEAR FIRST SEMESTER EXAMINATION FOR DEGREE IN BACHELOR OF SCIENCE IN MATHEMATICS & PROGRAMMING
BACHELOR OF SCIENCE IN MATHEMATICS & COMPUTER
BACHELOR OF SCIENCE IN STATISTICS
BACHELOR OF EDUCATION (SCIENCE)
BACHELOR OF EDUCATION (ARTS)

SMA 335: ORDINARY DIFFERENTIAL EQUATIONS I

DATE: 15TH JANUARY, 2025 **TIME: 8.30 AM – 10.30 AM**

INSTRUCTIONS:
*Answer question **ONE (Compulsory)** and **any other TWO** questions*

QUESTION ONE *(COMPULSORY)* (30 MARKS)

a) Differentiate between Ordinary differential equation and Partial differential equation of a differential equations. (2 marks)

b) Find the value of k for which $y = e^{kx}$ is a solution to the differential equation

$y''' - 7y'' + 14y' - 8y = 0$ (4 marks)

c) Find the differential equation associated with the equation $y = Ax^2 + Bx + 2$ (5 marks)

d) Find the order and the degree of differential equation

i. $\sqrt{y + \left(\frac{dy}{dx}\right)^2} = x + 1$ (2 marks)

ii. $\left(\frac{d^2y}{dx^2}\right)^{\frac{3}{2}} = \frac{dy}{dx} + y + 2$ (2 marks)

e) A linear equation of order 3 is given as ; $\frac{d^3y}{dx^3}-\frac{d^2y}{dx^2}-4\frac{dy}{dx}+4y=e^{3x}$, determine its auxiliary equation and hence the general solution (5marks)

f) Consider the DE ; $\frac{d^2y}{dx^2}-5\frac{dy}{dx}+6y=0$, show that its solutions are independent using the Wroskian and generate the general solution (5marks)

g) Reduce the equation $y''''+y=0$ into a system of four linear equations. (5marks)

QUESTION TWO (20 MARKS)

a) Define:
   i) Homogeneous equation (3marks)
   ii) Non-linear equation (3marks)

b) Solve the Initial value problem; (8marks)

$$\frac{d^2y}{dx^2}-6\frac{dy}{dx}+25y=0, y(0)=-3, y'(0)=-1$$

c) Given that $x^2y'+xy=1, x>0, y(1)=2$,use the method of Integrating factors to solve the DE (6marks)

QUESTION THREE (20 MARKS)

a) Solve using power series $y''-y=x$ at $x=0$ (10marks)

b) Determine the complementary function, the particular integral and hence the general solution of; $\frac{d^4y}{dx^4}+2\frac{d^3y}{dx^3}+\frac{d^2y}{dx^2}=\sin 3x$ (10marks)

QUESTION FOUR (20 MARKS)

a) Show that $y=C_1\sin x+C_2x$ is a solution of the differential equation $(1-xCotx)y''-xy'+y=0$ (5 marks)

b) Find the differential equation associated with the equation $y^2=Ax^2+Bx+C$ (5 marks)

c) Using method of separation of variables, solve the differential equation $(xy^2 + x)dx + (yx^2 + y)dy = 0$ (10 marks)

QUESTION FIVE (20 MARKS)

a) Solve the Bernoulli equation $\frac{dy}{dx} + 3x^2y = x^2y^3$ (10 marks)

b) Find the orthogonal trajectories of the family of curves $x^2 + y^2 = C^2$ (5 marks)

c) Show that $(y\cos x + 2xe^y)dx + (\sin x + x^2e^y - 1)dy = 0$ is exact and hence solve (5marks)