



MACHAKOS UNIVERSITY

University Examinations 2020/2021 Academic Year

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

FIRST YEAR SECOND SEMESTER EXAMINATION FOR

BACHELOR OF SCIENCE (ACTUARIAL SCIENCE)

SST 103: LINEAR ALGEBRA

DATE: 22/6/2021

TIME: 2.00-4.00 PM

INSTRUCTIONS:

Answer ALL the questions in Section A and ANY TWO Questions in Section B

SECTION A

QUESTION ONE 30 Marks (Compulsory)

a) Show that $(A - B)(A + B) = A^2 - B^2$ iff $AB = BA$ i.e A & B commute (4 marks)

b) Given $A = \begin{pmatrix} 1 & 2 \\ 3 & 5 \end{pmatrix}$, $B = \begin{pmatrix} 2 & 1 \\ 1 & 3 \end{pmatrix}$, show that $(AB)^T = B^T A^T$. (3 marks)

c) Determine the values of x for which the determinant is zero where

$$\begin{vmatrix} x-2 & 4 & 3 \\ 1 & x+1 & -2 \\ 0 & 0 & x-4 \end{vmatrix} \quad (4 \text{ marks})$$

d) Reduce $\begin{bmatrix} 3 & 4 & -1 & 1 \\ 1 & -1 & 3 & 1 \\ 4 & -3 & 11 & 2 \end{bmatrix}$ to reduced row-echelon (canonical) form. (3 marks)

e) Find the angle between $u = i + j + k$ and $v = i + j - k$ (3 marks)

f) Solve the following simultaneous equation using inverse matrix method

$$2x - y = 4$$

$$3x + 2y = 6 \quad (3 \text{ marks})$$

- g) If $\left| \frac{y}{y^2 + 1} \right| = 4$ find y (3 marks)
- h) Show that the vector $(11, 3, -8)$ is a linear combination of the $(1, 1, 0)$ and $(2, 1, -1)$ (4 marks)
- i) Calculate the x and y values for the vector $(x, y, 1)$ that is orthogonal to the vectors $(3, 2, 0)$ and $(2, 1, -1)$. (3 marks)

SECTION B

QUESTION TWO (20 MARKS)

- a) Let $a = 3i - j + k$ and $b = i + 2j - k$
- Find $\vec{a} \times \vec{b}$
 - Show that $\vec{a} \times \vec{b}$ is orthogonal to $\vec{a}$
 - Show that $\vec{a} \times \vec{b}$ is orthogonal to $\vec{b}$ (6 marks)
- b) Let $A = \begin{bmatrix} 2 & -5 & 2 \\ 1 & 2 & -4 \\ 3 & -4 & -6 \end{bmatrix}$
- Find $\det A$
 - Find $\text{Adj}(A)$
 - Hence find A^{-1} (6 marks)
- c) Find the cross product of the following vectors $(2, 4, -1)$ and $(3, 6, 1)$ (4 marks)
- d) Reduce the following matrixes to be reduced echelon form
- $$A = \begin{pmatrix} 3 & 4 & -1 & 1 \\ 1 & -1 & 3 & 2 \\ 4 & -3 & 11 & 2 \end{pmatrix} \quad (4 \text{ marks})$$

QUESTION THREE (20 MARKS)

- a) Solve using Cramer's rule.

$$x_1 + 3x_2 + 2x_3 = 3$$

$$2x_1 + 4x_2 + 2x_3 = 8$$

$$x_1 + 2x_2 - x_3 = 10$$

(5 marks)

- b) Solve the following equations using inverse matrix method

$$\begin{aligned}x + 2y + z &= 4 \\ 3x - 4y - 2z &= 2 \\ 5x + 3y + 5z &= -1\end{aligned}$$

(5 marks)

- c) Solve for x, y and z using Gauss-Jordan method

$$\begin{aligned}x + y + z &= 5 \\ x + 2y + 3z &= 10 \\ 2x + y + z &= 6\end{aligned}$$

(5 marks)

- d) Solve by elimination method

$$\begin{aligned}x_1 + x_2 + x_3 &= 5 \quad \dots \\ x_1 + 2x_2 + 3x_3 &= 10 \dots \\ 2x_1 + x_2 + x_3 &= 0 \dots\end{aligned}$$

(5 marks)

QUESTION FOUR (20 MARKS)

- a) Define a vector space (6 marks)
- b) Show that $W = \{(x, y) / x = 2y\}$ is a subspace for $\mathbb{R}^2$. (4 marks)
- c) Prove that the diagonals of a rhombus are perpendicular. (5 marks)
- d) Find the parametric and the symmetric equations of the line passing through the point $(2, 3, -4)$ and parallel to the vector $(3, 5, -6)$ (5 marks)

QUESTION FIVE (20 MARKS)

- a) Show that $w = \{(x, y, z) | x + y + z = 0\}$ is a subspace of $\mathbb{R}^3$ (5 marks)
- b) Calculate the value of k for the vectors $\vec{u} = (1, k)$ and $\vec{v} = (-4, k)$ knowing that they are orthogonal. (5 marks)
- c) Show that the vectors $u=(1,-1,0)$ $v=(1,3,-1)$ and $w=(5,3,-2)$ are linearly dependent. (5 marks)
- d) Find the rank of the following matrix

$$\begin{bmatrix} 2 & 4 & 3 \\ 3 & -3 & 2 \\ 2 & 0 & 3 \end{bmatrix}$$

(5 marks)