MACHAKOS UNIVERSITY

University Examinations 2020/2021 Academic Year

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

THIRD YEAR SPECIAL /SUPPLEMENTARY EXAMINATION FOR

BACHELOR OF EDUCATION

SMA 363 TESTS OF HYPOTHESIS I

DATE: 26/3/2021 **TIME: 2.00-4.00 PM**

INSTRUCTION: *Answer question one and any other two Questions*

QUESTION ONE (30 MARKS)

a) Explain briefly what is meant by hypothesis testing as used in statistics. (2 Marks)

b) Outline the general procedure used for hypothesis testing. (4 Marks)

c) Differentiate between Type I and Type II errors as used in hypothesis testing. (4 Marks)

d) Let X be a binomial random variable. We wish to test the hypothesis $H_0: p = 0.8$ against $H_a: p = 0.6$. Let $\alpha = 0.03$ be fixed. Find β for $n = 20$. (4 Marks)

e) Use the Neyman-Pearson Lemma to obtain the best critical region for testing $\theta = \theta_0$ against $\theta = \theta_1 > \theta_0$, in the case of a normal population $N(\theta, \sigma^2)$, where σ^2 is known. (7 Marks)

f) The average IQ of a sample of 50 University students was found to be 105. Carry out a statistical test to determine whether the average IQ of University students is greater than 100, assuming that IQ's are normally distributed. It is known from previous studies that the standard deviation of IQ's among students is approximately 20. (5 Marks)

g) Differentiate between a Most powerful test and the Uniformly most powerful test. (4 Marks)

QUESTION TWO (20 MARKS)

a) A random variable X is believed to follow an $Exp(\lambda)$ distribution. In order to test the null hypothesis $\mu = 20$ against the alternative $\mu = 30$, where $\mu = \frac{1}{\lambda}$, a single value is observed from the distribution. If this value is less than 28, H_0 is accepted, otherwise H_0 is rejected. Find the probabilities of

i. a Type I error

ii. a Type II error (8 Marks)

b) Let $X_1, \ldots, X_n$ be a random sample from a Poisson distribution with mean λ. Derive the most powerful test for testing $H_0: \lambda = 3$ against $H_a: \lambda = 6$. (7 Marks)

c) Let $X_1, \ldots, X_n$ be a random sample from a $U(0, \theta)$ distribution. Find the most powerful α-level test for testing $H_0: \theta = \theta_0$ versus $H_a: \theta = \theta_1$, where $\theta_0 < \theta_1$. (5 Marks)

QUESTION THREE (20 MARKS)

a) Let p be the probability that a coin will fall head in a single toss. In order to test the hypothesis $H_0: p = \frac{1}{2}$, the coin is tossed 5 times and the hypothesis H_0 is rejected if more than 3 heads are obtained. Find the probability of Type I error. If $H_a: p = \frac{3}{4}$, find the probability of Type II error.

(6 Marks)

b) Let X have a p.d.f. of the form

$$f(x, \theta) = \frac{1}{\theta} \exp\left(-{}^{x}/_{\theta}\right), 0 < x < \infty, \theta > 0$$
$$= 0, elsewhere$$

To test $H_0: \theta = 2$ against $H_a: \theta = 1$ use a random sample X_1, X_2 of size 2 and define a critical region $C = \{(x_1, x_2): 9.5 \leq x_1 + x_2\}$. Determine the power of the test. (4 Marks)

c) Let $X_1, \ldots, X_n$ be a random sample from a $N(\mu, \sigma^2)$. Assume that σ^2 is known. Determine an appropriate likelihood ratio test to test $H_0: \mu = \mu_0$ vs $H_a: \mu \neq \mu_0$ at level α. (10 Marks)

QUESTION FOUR (20 MARKS)

a) Define 'Likelihood ratio Test'. Under what circumstances would you recommend this test? (4 Marks)

b) Let $x_1, \ldots, x_n$ be a random sample from a normal distribution $N(\theta_1, \theta_2)$. Use the likelihood ratio test to obtain the best critical region of size α under $H_0: \theta_1 = 0$ against $H_1: \theta_1 \neq 0$. (8 Marks)

c) It is desired to investigate the level of premium charged by two companies for contents policies for houses in a certain area. Random samples of 10 houses insured by Company A are compared with 10 similar houses insured by Company B. Premiums charged in each case are as follows:

Company A	117	154	166	189	190	202	233	263	289	331
Company B	142	160	166	188	221	241	276	279	284	302

For these data: $\sum A = 2{,}134$, $\sum A^2 = 494{,}126$, $\sum B = 2{,}259$, $\sum B^2 = 541{,}463$. Test at 5% significance level, whether the level of premiums charged by Company B are higher than those charged by Company A. (8 Marks)

QUESTION FIVE (20 MARKS)

a) In order to increase the efficiency with which employees in a certain organisation can carry out a task, 5 employees are sent on a training course. The time in seconds to carry out the task both before and after the training course is given below for the 5 employees:

Employee	1	2	3	4	5
Before	42	51	37	43	45
After	38	37	32	40	48

Formulate the hypothesis and test whether the training course has had the desired effect. Take $\alpha = 0.05$. (8 Marks)

b) Use the method of least squares to fit a straight line to the data points below:

x	-1	0	2	-2	5	6	8	11	12	-3
y	-5	-4	2	-7	6	9	13	21	20	-9

i. Estimate β_0 and β_1. (4 Marks)

ii. Test the following hypothesis at 5% significance level

$$H_0: \beta_1 = 2 \; Vs \; H_1: \beta_1 \neq 2$$

and interpret the result obtained. (8 Marks)