



MACHAKOS UNIVERSITY

University Examinations 2022/2023 Academic Year

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

THIRD YEAR SECOND SEMESTER EXAMINATION FOR

BACHELOR OF SCIENCE (AUCTORIAL SCIENCE)

BACHELOR OF EDUCATION (SCIENCE)

BACHELOR OF EDUCATION (ARTS)

SMA 330: NUMERICAL ANALYSIS I/SCO213: NUMERICAL ANALYSIS FOR
COMPUTER SCIENCE

DATE: 11/4/2023

TIME: 11:00 – 1:00 PM

INSTRUCTIONS:

Answer **Question One** and **Any Two** Other Questions

QUESTION ONE (COMPULSORY) (30 MARKS)

- a) Show that the operators Δ and E commute. (2 marks)
- b) If 0.182 is the approximate value of $\frac{2}{11}$ find the absolute, relative and percentage errors. (4 marks)
- c) Use Lagrange's formula to prove that $y_0 = \frac{1}{2}(y_1 + y_{-1}) - \frac{1}{16}[(y_3 - y_1) - (y_{-1} - y_{-3})]$ (5 marks)
- d) Solve $x^3 - 9x + 1 = 0$ for the root between $x = 2$ and $x = 4$, by bisection method. (5 marks)
- e) How accurately should the length and the time of vibration of a pendulum should be measured in order that the computed value of g be correct to 0.01% (7 marks)

QUESTION TWO (20 MARKS)

- a) Prove that the forward difference of the quotient of two functions is given by

$$\Delta \left(\frac{f(x)}{g(x)} \right) = \frac{g(x)\Delta f(x) - f(x)\Delta g(x)}{g(x+h)g(x)} \quad (5 \text{ marks})$$

- b) Set up a Newton iteration for computing the square root of a given positive number. Using the same find the square root of 2 exact to six decimal places. (7 marks)
- c) Find a root of the equation $x^3 + x^2 - 1 = 0$ on the interval $[0,1]$ with an accuracy of 10^{-4} (8 marks)

QUESTION THREE (20 MARKS)

- a) Construct the forward difference table, where $f(x) = \frac{1}{x}$, $x = 1(0.2)2,4D$ (4 marks)
- b) Find $\int_0^{10} \frac{1}{1+x^2} dx$ using Simpson's one third rule. (7 marks)
- c) Find the interpolating polynomial by Newton's divided formula for the following table and then calculate $f(2.1)$.

x	0	1	2	4
$f(x)$	1	1	2	5

(9 marks)

QUESTION FOUR (20 MARKS)

- a) Using Aitken's scheme and the following values evaluate $\log_{10} 301$

x	300	304	305	307
$\log_{10} x$	2.4771	2.4829	2.4843	2.4871

(4 marks)

- b) Find the positive solution of the transcendental equation $2\sin x = x$ (5 marks)
- c) Find the cubic polynomial which takes the following values;
 $f(1)=24$, $f(3)=120$, $f(5)=336$, and $f(7)=720$. hence or otherwise, obtain the value of $f(8)$. (5 marks)

- d) Evaluate $\int_0^1 e^{-x^2} dx$ by means of Trapezoidal rule with $n = 10$ (6 marks)

QUESTION FIVE (20 MARKS)

- a) Derive the Newton-Raphson iterative formula

$$x_n = x_{n-1} - \frac{f(x_{n-1})}{f'(x_{n-1})}, n \geq 1$$

For the solution of the non-linear equation $f(x) = 0$. (10 marks)

- b) Use the method in a.) to find the root of the equation $\tan x = x$ near $x = 4.5$ correct to four decimal places. (10 marks)