MACHAKOS UNIVERSITY

University Examinations 2020/2021 Academic Year

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

FOURTH YEAR FIRST SEMESTER EXAMINATION FOR

BACHELOR OF SCIENCE (MATHEMATICS AND COMPUTER SCIENCE)

BACHELOR OF EDUCATION (SPECIAL NEED EDUCATION)

BACHELOR OF SCIENCE (MATHEMATICS)

BACHELOR OF EDUCATION (SCIENCE)

BACHELOR OF EDUCATION (ARTS)

SMA 430: NUMERICAL ANALYSIS II

DATE: 18/8/2021 **TIME: 2.00-4.00 PM**

INSTRUCTIONS:

ANSWER QUESTION ONE (COMPULSORY) ANY OTHER TWO QUESTIONS

QUESTION ONE (30 MARKS)

a) Highlight any five direct methods of solving linear systems of equations (5 marks)

b) Highlight any two iterative methods of solving linear systems of equations. (2 marks)

c) Solve the following system of equations using Gaussian elimination

$$\begin{bmatrix} 8 & 1 & 8 \\ 9 & 1 & 2 \\ 5 & 2 & 8 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 49 \\ 33 \\ 52 \end{bmatrix}$$ (5 marks)

d) Determine the condition number for the system $\begin{bmatrix} 1.8 & 2.1 \\ 5.3 & 6.2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2.1 \\ 6.2 \end{bmatrix}$ (5 marks)

e) Given the matrix $\begin{bmatrix} 8 & 2 & -2 \\ 2 & -8 & 3 \\ 2 & 1 & 9 \end{bmatrix}$, obtain the Jacobi's iterative matrix. (4 marks)

f) Obtain the spectral radius of matrix given by $A = \begin{bmatrix} 3 & 2 & 0 \\ 2 & -1 & -10 \\ 7 & 4 & -2 \end{bmatrix}$ (5 marks)

g) Solve the following system of equations using Gauss-Jordan method

$$\begin{bmatrix} 1 & 3 & 2 \\ 1 & 2 & 3 \\ 2 & -1 & 4 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 17 \\ 16 \\ 13 \end{bmatrix}$$ (4 marks)

QUESTION TWO (20 MARKS)

a) Determine the Euclidean norm of the matrix A, where $A = \begin{bmatrix} -4 & 7 & 1 \\ 9 & -4 & 4 \\ 3 & -1 & 12 \end{bmatrix}$ (4 marks)

b) Solve the following system of equations using Cholesky's factorization method

$$2x - 6y + 8z = 24$$
$$5x + 4y - 3z = 2$$
$$3x + y + 2z = 16$$
(7 marks)

c) Using Euler's algorithm, solve the equation $\frac{dy}{dx} = 1 - y$, with the initial condition $x = 0,\ y = 0$ and tabulate the solutions at $x = 0.1, 0.2$ and 0.3 (9 marks)

QUESTION THREE (20 MARKS)

a) Given the following systems of equations

$$4x + y + 2z = 4$$
$$3x + 5y + z = 7$$
$$x + y + 3z = 3$$

i) Obtain the exact solution for the system (4 marks)

ii) Set up the Jacobi iterative scheme for the solution , iterate three times starting with the initial vector $x = y = z = 0$ (6 marks)

b) Using Chebyshev polynomials, obtain the least squares approximation of second degree for $f(x) = x^4$ on $[-1,1]$ (5 marks)

c) Given the Rodrique's formula $p_n(x) = \frac{1}{2^n n!}\frac{d^n}{dx^n}(x^2-1)^n$, obtain the first three (3) Legendre polynomials. (5 marks)

QUESTION FOUR (20 MARKS)

a) Solve the following system of equations using Crout's method

$$\begin{bmatrix} 1 & 1 & 1 \\ 2 & -3 & 4 \\ 3 & 4 & 5 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 9 \\ 13 \\ 40 \end{bmatrix}$$ (7 marks)

b) Find the Least squares approximation of second degree for the discrete data (6 marks)

X	0.1	0.2	0.3	0.4	0.5	0.6
Y	21	11	7	6	5	6

c) Using Runge-Kutta method, find an approximate value of y for $x = 0.3$, if

$\frac{dy}{dx} = x + y^2$ given that $y = 1.0$ when $x = 0$ (7 marks)

QUESTION FIVE (20 MARKS)

a) Determine the solution of the following system of equations using Gauss-Seidal iteration method

$$8x_1 + 2x_2 - 2x_3 = 8$$
$$x_1 - 8x_2 + 3x_3 = -4$$
$$2x_1 + x_2 + 9x_3 = 12$$

(10 marks)

b) Given $\frac{dy}{dx} = 1 + xy$ with initial condition that $y = 1,\ x = 0,$ Compute $y(0.1)$ correct to four decimal places by using Taylor's series second order method with step length $h = 0.05$

(10 marks)