MACHAKOS UNIVERSITY

UNIVERSITY SUPPLEMENTARY EXAMINATION FOR THE DEGREE OF BACHELOR OF SCIENCE ELETRICAL AND TELECOMMUNICATION, MECHANICAL AND MANUFACTURING, CIVIL AND BUILDING ENGINEERING

ECU 106: ENGINEERING MATHEMATICS III

DATE: APRIL 2020 TIME 2 HOURS

INSTRUCTIONS: ANSWER QUESTION ONE (COMPULSORY) ANY OTHER TWO QUESTIONS

QUESTION ONE (30 MARKS)

a) A force $15 units$ acts in the direction of the vector $i-2j+2k$ and passes through a point $2i-2j+2k$. Find the moment of the force about the point $i+j+k$ **(5 marks)**

b) If $A=\begin{bmatrix}-2 & 2 & -3\\ 2 & 1 & -6\\ -1 & -2 & 0\end{bmatrix}$, write the characteristic equation, hence obtain the characteristic root of the matrix A **(5 marks)**

c) If $A=\begin{bmatrix}1 & 2 & 2\\ 2 & 1 & 2\\ 2 & 2 & 1\end{bmatrix}$, show that $A^2-4A-5I=0$, where I, 0 are the unit matrix and the null matrix of order three respectively. Use this result to find A^{-1} **(5 marks)**

d) Given $a=2i+3j-k$; $b=i-j-2k$; $c=i+2j+2k$. Show that $a\times b\times c=(a.c)b-(a.b)c$ **(5 marks)**

e) The sum of the three numbers is 6. I we multiply the third number by 2 and add the first number to the result, we get 7. By adding second and third numbers to three times the first number we get 12. Use determinants to find the numbers **(5 marks)**

f) If $A=\begin{bmatrix}1 & 2\\ -2 & 3\end{bmatrix}$; $B=\begin{bmatrix}2 & 1\\ 2 & 3\end{bmatrix}$ and $C=\begin{bmatrix}-3 & 1\\ 2 & 0\end{bmatrix}$. Verify that:-

i. $(AB)C=A(BC)$ **(3 marks)**

ii. $A(B+C)=AB+AC$ **(2 marks)**

QUESTION TWO (20 MARKS)

a) Find the unit vector perpendicular to both $a+b$ and $b+c$ where $a=i+j-k$; $b=-i+2j+k$; and $c=i+2j+k$ **(4 marks)**

b) The position vectors of the points A, B, C and D are $3i-2j-k$; $2i+3j-4k$, $-i+j+2k$; $4i+5j+\lambda k$ respectively. If the four points lie in a plane find the value of n λ **(4 marks)**

c) If $A+B=\begin{bmatrix}5 & 3 & 1\\ 2 & 4 & 6\\ 1 & 0 & 2\end{bmatrix}$ and $A-B=\begin{bmatrix}1 & 7 & 5\\ 0 & 2 & 4\\ 5 & 4 & 2\end{bmatrix}$ find A^T and B^T **(4 marks)**

d) Determine the value of a and b for which the system

$$\begin{bmatrix}3 & -2 & 1\\ 5 & -8 & 9\\ 2 & 1 & a\end{bmatrix}\begin{bmatrix}x\\ y\\ z\end{bmatrix}=\begin{bmatrix}b\\ 3\\ -1\end{bmatrix}$$

i. Has a unique solution **(4 marks)**

ii. Has no solution **(2 marks)**

iii. Has infinitely many solutions **(2 marks)**

QUESTION THREE (20 MARKS)

a) Find the value of ρ for which the vectors $4i+\rho j+2k$; $-i+4j+3k$ and $8i+j-3k$ are coplanar **(3 marks)**

b) Find the area of a parallelogram having diagonals $3i+j-2k$ and $i-3j+4k$ **(4 marks)**

c) Solve the linear system by Gaussian elimination method:-

$$x-y+2z=3$$
$$x+2y+3z=5$$
$$3x-4y-5z=-13$$

(5 marks)

d) If $AX=B$, solve using matrix method given that:-

$A\equiv\begin{bmatrix}1 & 2 & 3\\ 3 & -1 & 1\\ 4 & 2 & 1\end{bmatrix}$; $X=\begin{bmatrix}x\\ y\\ z\end{bmatrix}$; and $B=\begin{bmatrix}1\\ 2\\ 3\end{bmatrix}$ **(8 marks)**

QUESTION FOUR (20 MARKS)

a) Find the vector of $\sqrt{201}$ which is perpendicular to $2i-j+k$ and $i-3j-5k$

(4 marks)

b) A force of magnitude $5units$ acting parallel to $2i-2j+k$ displaces the point of application from $(1,2,3)$ to $(5,3,7)$. Find the work done **(4 marks)**

c) Determine the nature of solutions of the linear system

$$5x+3y+7z=4$$
$$3x+26y+2z=9$$ Hence solve it **(6 marks)**
$$7x+2y+10z=5$$

d) Use elementary transformation to find the inverse of the matrix

$$A=\begin{bmatrix}1 & 3 & 3\\ 1 & 4 & 3\\ 1 & 3 & 4\end{bmatrix}$$ **(6 marks)**

QUESTION FIVE (20 MARKS)

a) If $a=-i+j+k$ and $b=7i+2j+k$, find the projection of a in the direction of b

(4 marks)

b) Diagonalize the matrix $A=\begin{bmatrix}3 & 1 & 4\\ 0 & 2 & 6\\ 0 & 0 & 5\end{bmatrix}$ **(16 marks)**