MACHAKOS UNIVERSITY
University Examinations for 2024/2025 Academic Year

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS, STATISTICS AND ACTUARIAL SCIENCES

SECOND YEAR FIRST SEMESTER EXAMINATION FOR DEGREE IN

BACHELOR OF SCIENCE STATISTICS AND COMPUTING
BACHELOR OF SCIENCE METHEMATICS
BACHELOR OF EDUCATION (SCIENCE)
BACHELOR OF EDUCATION (ARTS)

SMA 230: VECTOR ANALYSIS

DATE: 22ND JANUARY, 2025
TIME: 8.30 AM – 10.30 AM

INSTRUCTIONS

Answer Question One (Compulsory) and Any Other Two Questions

QUESTION ONE (30 MARKS)

a) Find the unit vector paralleled to the resultant vectors;

$\vec{r}_1 = 2\hat{i} + 4\hat{j} - 5\hat{k}$ and

$\vec{r}_2 = \hat{i} + 2\hat{j} + 3\hat{k}$

(3 marks)

b) If $\underset{\sim}{\boldsymbol{a}}$ and $\underset{\sim}{\boldsymbol{b}}$ are non-collinear vectors and

$\vec{A} = (x + 4y)\underset{\sim}{\boldsymbol{a}} + (2x + y)\underset{\sim}{\boldsymbol{b}}$ and

$\vec{B} = (y - 2x + 2)\underset{\sim}{\boldsymbol{a}} + (2x - 3y - 1)\underset{\sim}{\boldsymbol{b}}$

Find x and y such that

$3\vec{A} = 2\vec{B}$

(4 marks)

c) Find the projection of the vector $\vec{A} = \hat{i} - 2\hat{j} + \hat{k}$ on the vector $\vec{B} = 4\hat{i} - 4\hat{j} + 7\hat{k}$

(4 marks)

d) Find the volume of a parallelepiped whose edges are represented by the vectors $\vec{A} = 2\hat{i} - 3\hat{j} + 4\hat{k}$, $\vec{B} = \hat{i} + 2\hat{j} - \hat{k}$ and $\vec{C} = 3\hat{i} - \hat{j} + 2\hat{k}$ if $\vec{A}$, $\vec{B}$ and $\vec{C}$ do not form a right–handed system.

(4 marks)

e) If $\vec{A} = 5\hat{i} - 2\hat{j} + 3\hat{k}$,

$\vec{B} = 3\hat{i} + \hat{j} - 2\hat{k}$ and

$\vec{C} = \hat{i} - 3\hat{j} + 4\hat{k}$.

Determine the vector triple products;

i. $\vec{A}X(\vec{B}X\vec{C})$ (4 marks)

ii. $(\vec{A}X\vec{B})X\vec{C}$ (4 marks)

f) Evaluate

$\int_V \vec{F}\, dV$ where V is the region bounded by the planes $x = 0$, $x = 2$

$y = 0, y = 3, z = 0, z = 4$

and $\overrightarrow{F} = xy\hat{i} + z\hat{j} - x^2\hat{k}$

(7 marks)

QUESTION TWO (20 MARKS)

a) A particle moves along a curve whose parametric equations are

$x = e^{-3t}$, $y = 2cos3t$,

$z = 4sin3t$ Where t is the time.

Determine;

i. Its velocity and acceleration at any time (2 marks)

ii. The magnitudes of the velocity and acceleration at $t = 0$ (2 marks)

b) Verify Stoke's theorem for $\vec{A} = (2x - y)\,\hat{i} - yz^2\,\hat{j} - y^2z\,\hat{k}$, where S is the upper half surface of the sphere $x^2 + y^2 + z^2 = 1$ and C is its boundary (12 marks)

c) Given the vector $\vec{a}' = \frac{\vec{b}\times\vec{c}}{\vec{a}\cdot\vec{b}\times\vec{c}}$,

$\vec{b}' = \frac{\vec{c}\times\vec{a}}{\vec{a}\cdot\vec{b}\times\vec{c}}$ and $\vec{c}' = \frac{\vec{a}\times\vec{b}}{\vec{a}\cdot\vec{b}\times\vec{c}}$.

Show that if ;

$\vec{a} \cdot \vec{b} \times \vec{c} \neq 0,$

If $\vec{a} \cdot \vec{b} \times \vec{c} = V,$

then $\vec{a}' \cdot \vec{b}' \times \vec{c}' = \frac{1}{V}$ (4 marks)

QUESTION THREE (20 MARKS)

a) Evaluate

$\iint_S \vec{F} . \hat{n} dS$ over the entire surface of the region bounded by the cylinder $x^2 + z^2 = 9$, $x = 0$, $y = 0$, $y = 8$, $z = 0$ if $\vec{F} = 6z\hat{i} + (2x + y)\hat{j} - x\hat{k}$

(8 marks)

b) Find the area of the upper cap cut

from the sphere $x^2 + y^2 + z^2 = 2$ by the cylinder $x^2 + y^2 = 1$

(8 marks)

c) Evaluate

$$\oint_C (2x - y)dx + (2y + x)dy$$

around the boundery C of the ellipse $x^2 + 9y^2 = 16$

(4 marks)

QUESTION FOUR (20 MARKS)

a) Find the total work done in

moving a particle in a force field given by

$$\vec{F} = 3xy\hat{i} - 5z\hat{j} + 10x\hat{k}$$

along the curve

$$x = t^2 + 1, y = 2t^2, z = t^3$$

from

$$t = 1 \; to \; t = 2$$

(6 marks)

b) If $\emptyset(x, y, z) = 2xz^4 - x^2y$, find $\vec{\nabla}\phi$ and $|\vec{\nabla}\phi|$ at the

point (2,-2,-1)

(4 marks)

c) Given the space curve

$x = t^2 + 1, y = 4t - 3, z = 2t^2 - 6t$

find

i. The unit tangent (2 marks)

ii. The principal normal (2 marks)

iii. Curvature (2 marks)

iv. Radius of curvature (2 marks)

v. The binormal (2 marks)

QUESTION FIVE (20 MARKS)

a) If $\vec{A} = x^2y\,\hat{i} - 2xz\,\hat{j} + 4xz^2\,\hat{k}$,
find curl curl $\vec{A}$

(4 marks)

b) If $\emptyset(x, y, z) = xy^2z$ and
$\vec{A} = xz\,\hat{i} - xy^2\,\hat{j} + yz^2\,\hat{k}$
find $\frac{\partial^3\,(\phi\,\vec{A})}{\partial x^2\,\partial z}$ at the point (2,-1,1)

(4 marks)

c) Use divergence theorem to evaluate the surface integral

$$\iint_S \vec{A} \cdot \hat{n}\, dS$$

where $\vec{A} = 4x\hat{i} - 2y\hat{j} + z^2\hat{k}$
and S is the surface enclosed by
$x^2 + y^2 = 4$, $z = 0$ and $z = 3$

(12 marks)