MACHAKOS UNIVERSITY

UNIVERSITY EXAMINATIONS -2017/2018

BACHELOR OF SCIENCE IN MATHS & COMP. SCIENCE AND BACHELOR OF SCIENCE IN STATISTICS AND PROGRAMMING, SECOND YEAR

SMA 230: VECTOR ANALYSIS

DATE: DECEMBER 2017 **TIME: 2 HRS**

INSTRUCTIONS: Answer Question ONE and any other TWO.

QUESTION ONE (30 MARKS)

a) If two vectors $\vec{A} = 2\hat{i} + \sqrt{3}\hat{j} - 3\hat{k}$, $\vec{B} = 6\hat{i} - 3\hat{j} + 2\hat{k}$ are inclined to each other at an angle θ. Find the value of θ. (4 mks)

b) Find the area of the triangle with vertices at $A(1,3,2)$, $B(2,-1,1)$, $C(-1,2,3)$. (5 mks)

c) Find the projection of the vector $7\hat{i} + 5\hat{j} + 2\hat{k}$ on the line passing though the points $A(2,3,-1)$, $B(-3,-5,4)$ (4 mks)

d) Given the position vector of any point on a curve is $\vec{r}(s) = x(s)\hat{i} + y(s)\hat{j} + z(s)\hat{k}$, show that $\frac{d\vec{r}}{ds} \bullet \frac{d^2\vec{r}}{ds^2} \times \frac{d^3\vec{r}}{ds^3} = \frac{\tau}{\rho^2}$ (5 mks)

e) Find the directional derivative of $\phi(x,y,z) = 4xz^3 - 3x^2y^2z$ at $(1,1,1)$ in the direction $3\hat{i} - 2\hat{j} + 5\hat{k}$ (4 mks)

f) Given the scalar function $\phi(x,y,z) = xy + y^2z$ and the curve C defined by $x = t^2$, $y = 2t$, $z = t + 5$ between the points $A(0,0,5)$, $B(1,2,6)$

i) Transform the integral $\oint_C \phi\, d\vec{r}$ in terms of an integral involving t only (3 mks)

ii) Determine the interval of parameter t from the given parametric equations (2 mks)

iii) Lastly, evaluate $\oint_C \phi\, d\vec{r}$ along the given curve (3 mks)

QUESTION TWO (20 MARKS)

a) Prove that $\vec{A} \times (\vec{B} \times \vec{C}) = \vec{B}(\vec{A} \bullet \vec{C}) - \vec{C}(\vec{A} \bullet \vec{B})$ for $\vec{A} = (A_1, A_2, A_3)$, $\vec{B} = (B_1, B_2, B_3)$, $\vec{C} = (C_1, C_2, C_3)$ (7 mks)

b) Given $\vec{A} = 3\hat{i} + 5\hat{j} + 2\hat{k}$, $\vec{B} = \hat{i} + 3\hat{j} - 2\hat{k}$, $\vec{C} = -\hat{i} + 5\hat{j} + 2\hat{k}$ illustrate that $(\vec{A} \times \vec{B}) \times \vec{C} \neq \vec{A} \times (\vec{B} \times \vec{C})$. (5 mks)

c) Find a set of vectors reciprocal to the set $2\hat{i}+3\hat{j}-\hat{k}$, $\hat{i}-\hat{j}-2\hat{k}$, $-\hat{i}+2\hat{j}+2\hat{k}$ (8 mks)

QUESTION THREE (20 MARKS)

a) If $\vec{A}=e^{-2t}\hat{i}+\ln(3t^2+1)\hat{j}-\tan t\hat{k}$, find $\frac{d\vec{A}}{dt}$, $\frac{d^2\vec{A}}{dt^2}$ at $t=0$ and their magnitudes. (6 mks)

b) Prove the Frenet- Seret relation:

i. $\frac{d\hat{B}}{ds}=-\tau\hat{N}$ (5 mks)

ii. $\frac{d\hat{N}}{ds}=\tau\hat{B}-k\hat{T}$ (2 mks)

c) Find $\hat{T}$, k, $\hat{N}$ i.e. the unit tangent vector, the curvature and the unit normal respectively for the curve $\vec{r}(t)=-3\sin 2t\hat{i}-3\cos 2t\hat{j}$ (6 mks)

QUESTION FOUR (20 MARKS)

a) Find an equation for the plane perpendicular to the vector $\vec{A}=6\hat{i}-3\hat{j}+2\hat{k}$ and passing through the terminal point $B(1,3,2)$ (4 mks)

b) Prove the following:

i. $\vec{\nabla}\bullet(\phi\vec{A})=(\vec{\nabla}\phi)\bullet\vec{A}+\phi(\vec{\nabla}\bullet\vec{A})$ for $\vec{A}=(A_1,A_2,A_3)$ and $\phi=\phi(x,y,z)$ (3 mks)

ii. $\vec{\nabla}\bullet(r^{-3}\vec{r})=0$ where $r=|\vec{r}|$ (3 mks)

c) Given that the curve $x=t$, $y=t^2$, $z=\frac{2t^3}{3}$, find the curvature and the binomial to the curve

(10 mks)

QUESTION FIVE (20 MARKS)

a) Determine the constants a, b, c such that

$\vec{V}=(x+cy+az)\hat{i}+(3x-y-bz)\hat{j}+(4x+2y+9z)\hat{k}$ is irrotational. (4 mks)

b) Evaluate $\iint_S \phi \hat{n} ds$ where $\phi = \frac{3}{8}xyz$ and S is the surface of the cylinder $x^2 + y^2 = 16$ between $z = 0$ and $z = 5$ in the first octant. (7 mks)

c) Verify the Green's theorem in the plane for $\oint_C (xy + y^2)dx + x^2 dy$, where C is the closed curve of the region bounded by $y = x$, $y = 2 - x^2$. (9 mks)