MACHAKOS UNIVERSITY

University Examinations for 2021/2022

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

THIRD YEAR SUPPLEMENTARY EXAMINATION FOR

BACHELOR OF SCIENCE (ACTUARIAL SCIENCE)

SAC 303: LIFE CONTIGENCIES II:

DATE: 18/3/2022 **TIME: 11:00 – 1:00 PM**

INSTRUCTIONS: Answer Question One and Any Other Two Questions

QUESTION ONE: 30 MARKS

a) Define the following actuarial notations.

i. Deferred annuities. (2 marks)

ii. Endowment assurance. (2 marks)

b) Using ELT 15 (males) mortality. Find the curtate expectation of life for :

i. A new born baby (2 marks)

ii. 21 year old actuarial student (2 marks)

c) Calculate the probabilities of the following events using the service table in the tables.

i. A member aged 42 exactly will retire through ill health in the year ending on his 60^{th} birthday. (2 marks)

ii. A member aged 45 exactly will withdraw from service by age 50. . . (3 marks)

d) Express $s_x(t)$ in its deterministic form. (3 marks)

e) In a certain population, the force of mortality equals 0.025 at all ages.

i. The probability that a life aged exactly 10 will die before age 12. (4 marks)

ii. The probability that a life aged exactly 5 will die between ages 10 and 12. (5 marks)

f) The random variable T_{xy} represents the time to failure of the joint life status (xy) . (x) is subject to a constant force of mortality of 0.01, and (y) is subject to a constant force of mortality of 0.02. calculate the value of $E(T_{xy})$ assuming that x and y are independent With respect to mortality. (5 marks)

QUESTION TWO (20 MARKS)

a) Claire aged exactly 30 buys a whole life assurance of £ 50000 payable at the end of the year of her death. Calculate the standard deviation of the P.V of the benefit using AM92 ultimate and 6% per annum interest. (5 marks)

b) A life office issues with-profit whole of life contracts, with the sum assured payable immediately on death of the life assured. Level premiums are payable monthly in advance to age 65 or until earlier death.

The life office markets two versions of this policy, one assumed to provide simple bonuses of 4% per annum of the sum assured vesting at the end of each policy year and the other assumed to provide compound bonuses of 4% of the sum assured, again vesting at the end of each policy year. The death benefit under each version does not include any bonus relating to the policy year of death.

The following basis is assumed to price these contracts:

- Mortality AM92 Select Interest 4% per annum
- Initial expenses £300
- Renewal expenses 2.5% of the second and subsequent monthly premiums
- Initial commission 50% of the gross annual premium
- Renewal commission 2.5% of the second and subsequent monthly premiums
- Claims expenses £250 at termination of the contract.

Calculate the level monthly premium required for each version of this policy issued to a life aged 30 exact at outset for an initial sum assured of £50,000. (15 marks)

QUESTION THREE (20 MARKS)

An employer wishes to introduce a lump-sum retirement benefit payable immediately on retirement at 65 or earlier other than on the grounds of ill-health. The amount of the benefit is £1,000 for each year of an employee s service, with proportionate parts of a year counting.

i) Give a formula to value this benefit for an employee currently aged x with n years of past service, defining all terms used. (10 marks)

ii) Using the Pension Scheme Tables from the Actuarial Formulae and Tables, calculate the value for an employee currently aged 30 exact with exactly 10 years past service. (4 marks)

iii) Calculate the level annual contribution payable continuously throughout this employee s service to fund the future retirement benefit (6 marks)

QUESTION FOUR (20 MARKS)

a) T_x And T_y are the complete future lifetimes of two lives aged x and y respectively: Let the random variable $g(T)$ take the following values.

$$g(T) = \begin{cases} \bar{a}_{\overline{T_x}|} & \text{if} \quad T_x \le T_y \\ \bar{a}_{\overline{T_y}|} & \text{if} \quad T_x > T_y \end{cases}$$

i. Describe the benefit which has present value equal to $g(T)$. (2 marks)

ii. Express $E[g(T)]$ as an integral. (2 marks)

iii. Write down an expression for the variance of $g(T)$ using assurance functions. (4 marks)

b) A life insurance company issues a joint life annuity to a male aged 68 and female aged 65. The annuity of £10,000 pa is payable annually in arears and continuous until both live have died. The insurance company values the benefit using PA92C20 mortality (males or females as appropriate) and 4% pa interest.

i. Calculate the expected present value and derive an expression for the variance of the present value in terms of appropriate single and life assurance functions. (8 marks)

ii. If the insurance company charges a premium of £ 150,000 for this policy. Calculate the probability that it makes a profit on the contract. (4 marks)

QUESTION FIVE (20 MARKS)

a) Proof that, (5 marks)

$$ {}_{n|m}q_x = \frac{l_{x+n} - l_{x+n+m}}{l_x} $$

b) A population is subject to a constant force of mortality of 0.015. Calculate:

i) The probability that a life aged 20 exact will die before age 21.25 exact. .

(2 marks)

ii) The curtate expectation of a life aged 20 exact (3 marks)

c) The following model of mortality was used in an investigation of the effects of where someone lives and income on the risk of death.

$$ \log_e \mu_x = \alpha + \beta_0 X + \beta_1 U + \beta_2 I $$

Where μ_x is the force of mortality at age x ,U takes the value 1. If the person lives in an urban area and 0 if the person lives in a rural area, I is annual income in us dollars and $\alpha, \beta_0, \beta_1 \; and \; \beta_2$ are parameters.

i. Explain why the Gompertz model is commonly used in investigations of human mortality. (2 marks)

ii. Show that the model is both a Gompertz model and a proportional hazards model. The estimates of parameters were $\alpha = -9.0$, $\beta_0 = 0.09$, $\beta_1 = 0.3 \; and \; \beta_2 = -0.0001$ (8 marks)