



MACHAKOS UNIVERSITY

UNIVERSITY SUPPLEMENTARY EXAMINATIONS

FOR THE DEGREE OF BACHELOR OF

SCIENCE IN COMPUTER SCIENCE

SCO 111: DIFFERENTIAL CALCULUS FOR COMPUTER SCIENCE

INSTRUCTION: Answer Question *ONE* which is compulsory and any other *TWO* Questions

QUESTION ONE (30 MARKS)

a.) Determine the $\lim_{x \rightarrow \infty} \frac{5x^2 - 3x + 2}{10x^2 - x + 100}$ (2 marks)

b.) Differentiate $x = t^3 + t^2$, $y = t^2 + t$ (3 marks)

c.) State the *L'Hôpital's Rule*, hence or otherwise evaluate

$$\lim_{x \rightarrow -3} \frac{x+3}{x^2+5x+6}$$
 (3 marks)

d.) Determine $\frac{dy}{dx}$ given that $y = \frac{\sin x}{x}$ (3 Marks)

e.) Find the derivative of the following function from the first principles.

$$f(x) = x^2$$
 (3 Marks)

f.) If $y = \frac{u^2 - 1}{u^2 + 1}$ and $u = \sqrt[3]{x^2 + 2}$ find $\frac{dy}{dx}$, (4 Marks)

g.) Obtain $\frac{df(x)}{dx}$ for $f(x) = \frac{\sin x + e^{2x}}{\sin x}$ (4 marks)

h.) Determine if the following function is continuous. $g(x) = \begin{cases} +1 & \text{if } x \geq 0 \\ -1 & \text{if } x < 0 \end{cases}$ (4 Marks)

i.) Write down the derivative of $\ln\left(\frac{1}{\sqrt{x}}\right)$ (4 marks)

QUESTION TWO (20 MARKS)

a.) Determine $\frac{dy}{dx}$ given that $x^2 + 2xy - 2y^2 + x = 2$, at the point $(-4, 1)$ (4 Marks)

b.) Differentiate $y = 3x^2 + \cos 2x$ from the first principles. (5 Marks)

c.) Find the derivative the following functions $y = x^2 - 4x$ and $x = \sqrt{2t^2 + 1}$, at $t = \sqrt{2}$ using chain rule. (5 Marks)

d.) Find the gradient of the curve $x = \frac{t}{1+t}$, and $y = \frac{t^3}{1+t}$ at the point $\left(\frac{1}{2}, \frac{1}{2}\right)$ (6 Marks)

QUESTION THREE (20 MARKS)

a.) Using logarithmic differentiation, differentiate the following functions;

i. $xe^x \sin x$ (3 Marks)

ii. $te^t \cos t$ (3 Marks)

b.) Evaluate $\lim_{x \rightarrow 0} \frac{\sqrt{2+x} - \sqrt{2}}{x}$ (4 Marks)

c.) Ink is dropped onto a blotting paper forming a circular stain which increases at the rate of $5\text{cm}^2 / \text{s}$. Find the rate of change of the radius when the area is 30cm^2 (5 Marks)

d.) Evaluate the derivative of the function given as $x^{\cos x}$ with respect to x , with respect to x , (5 marks)

QUESTION FOUR (20 MARKS)

a.) Differentiate $y = (x^2 + 1)^3 (x^3 + 1)^2$ (5 Marks)

b.) If $x = t^3, y = t^2 - t$ find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ (5 Marks)

c.) Find the maximum and minimum points of the curve $y = x^2(x+1) = x^3 + x^2$ (10 Marks)

QUESTION FIVE (20 MARKS)

a.) Find $\frac{dy}{dx}$ given that $x^2 + 2xy - 2y^2 + x = 2$ at the point $(-4, 1)$ (4 marks)

b.) Find the points on $y = x^4 + 4x^3 - 6$ at which the gradient is zero and determine the nature of the points (8 Marks)

c.) The parametric equations of a curve are $x = e^t$, $y = \sin t$, find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ as functions of t .

Hence show that $x^2 \frac{d^2y}{dx^2} + x \frac{dy}{dx} + y = 0$. (8 Marks)