MACHAKOS UNIVERSITY

University Examinations 2020/2021 Academic Year

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

SECOND YEAR …… SEMESTER EXAMINATION FOR

BACHELOR OF …………………………………………..

SMA 201: CALCULUS III

DATE: **TIME:**

INSTRUCTION:

Answer Question One and Any Other Two Questions

Mobile phones and any written material are prohibited in the examination room.

No writing should be done on this question paper. Any rough work should be done at the back of the answer booklet and canceled.

All answer booklets should be handed in at the end of the exam whether used or not.

Programmable calculators are prohibited

QUESTION ONE (COMPULSORY-30 MARKS)

a) (i) State Rolle's theorem (2 marks)

(ii) Using Rolle's theorem, show that the function $f(x) = x^3 - 4x$ is continuous and differentiable on R. (4 marks)

b) Find the value c guaranteed by the Cauchy's mean value theorem given the following function.

$f(x) = x^2, g(x) = x$ on $[1,4]$ (4 marks)

c) Evaluate the following limits using the L'Hospital rule.

(i) $\lim_{x\to 1} \frac{x^3-1}{4x^3-x-3}$ (2 marks)

(ii) $\lim_{x\to \frac{\pi}{2}} \frac{2x-\pi}{\cos x}$ (2 marks)

d) Use Maclaurin's theorem to expand the given function as a series of ascending powers of x.

$f(x) = \cos x$ (5 marks)

e) Find $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$ at (4,5) if $f(x,y) = x^2 + 3xy + y - 1$ (4 marks)

f) Work out the derivative of $f(x,y,z) = x^3 - xy^2 - z$ at $\rho_0(1,1,0)$ in the direction of $A = 2i - 3j + 6k$. (7 marks)

QUESTION TWO (20 MARKS)

a) Evaluate $\int_0^1 \int_0^1 \int_0^1 (x^2 + y^2 + z^2) dzdydx$ (6 marks)

b) Calculate the volume bounded by $f(x,y) = 1 - 6x^2y$ on the region R, for $0 \leq x \leq 2$, $-1 \leq y \leq 1$. (4 marks)

c) Verify both forms of Green's theorem for the field $F(x,y) = (x-y)i + xj$ and R bounded by the circle $c: R(t) = (\cos t)i + (\sin t)j, 0 \leq t \leq 2\pi$. (10 marks)

QUESTION THREE (20 MARKS)

a) The equation $xz + y\ln x + x^2 + 4 = 0$ defines x as a differentiable function of two independent variables y and z. Find $\frac{\partial x}{\partial y}$ and $\frac{\partial x}{\partial z}$ at the point (1,-1,3). (8 marks)

b) Find $(\frac{\partial w}{\partial x})_{y,z}$ and $(\frac{\partial w}{\partial x})_{t,z}$ if $w = x^2 + y - z + \sin t$ and $x + y = t$ (6 marks)

c) Find the value of $\frac{\partial f}{\partial t}$ at $t = -2$ for the function $f(x,y,z) = \sqrt{x^2 + y^2 + z^2}, x = 2 + t, y = -t - 1, z = t$ (6 marks)

QUESTION FOUR (20 MARKS)

a) i Prove that $\sin x = \sin a + \cos a\,(x-a) - \frac{\sin a\,(x-a)^2}{2!} - \frac{\cos c\,(x-a)^3}{3!}$ Where c is between a and x. (4 marks)

ii Use (a) (i) above to evaluate $\sin 51^0$ and estimate the error made. (6 marks)

b) Expand $\ln x$ as a series of ascending powers of $(x-a)$ upto the term containing $(x-a)^3$ and use it to evaluate $\ln 1.01$. (10 marks)

QUESTION FIVE (20 MARKS)

a) Find the equation of the tangent plane and the normal line of the curve $f(x,y,z) = x^2 + y^2 - 2xy - x + 3y - z = -4$ at $\rho_0(2,-3,18)$. (10 marks)

b) Use McLaurin's theorem to expand the given function as ascending powers of x upto the term x^5 (10 marks)