MACHAKOS UNIVERSITY

UNIVERSITY EXAMINATIONS FOR 2017/2018 ACADEMIC YEAR

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

THIRD YEAR SECOND SEMESTER EXAMINATION FOR

BACHELOR OF SCIENCE (MATHEMATICS AND COMPUTER SCIENCE)

BACHELOR OF SCIENCE (MATHEMATICS)

BACHELOR OF STATISTICS AND PROGRAMMING)

BACHELOR OF EDUCATION (SCIENCE)

BACHELOR OF EDUCATION (ARTS)

SMA466: MULTIVARIATE STATISTICAL METHODS II

DATE: 20/12/2017 **TIME: 8.30-10.30 AM**

INSTRUCTION: Answer Question **ONE** and Any Other **TWO** Questions

QUESTION ONE (COMPULSORY) (30 MARKS)

(a) (i) What is a mean vector (1 mark)

(ii) Describe how multivariate data are arranged (3 marks)

(b) The data below shows the scores of a sample of 15 students in mathematics, English and Kiswahili CATS in a certain school

$$\boldsymbol{X} = \begin{bmatrix} 4 & 8 & 6 & 8 & 9 \\ 8 & 7 & 4 & 4 & 10 \\ 10 & 9 & 7 & 5 & 9 \end{bmatrix}$$

Obtain

(i) Mean Vector (3 marks)

(ii) Variance-Covariance matrix (5 marks)

(iii)Correlation matrix (3 marks)

(c) A random sample of 10 was obtained from a bivariate normal population with mean vector $\boldsymbol{\mu}$ and a known variance-covariance matrix $\sum_0 = \begin{bmatrix} 4 & 4.2 \\ 4.2 & 9 \end{bmatrix}$. Test at $\alpha = 0.01$ level of significance for $H_0: \boldsymbol{\mu} = \boldsymbol{\mu}$ vs $H_1: \boldsymbol{\mu} \neq \boldsymbol{\mu_0}$ where $\boldsymbol{\mu_0} = (6,5)'$ and the sample mean vector is $\underline{\overline{X}} = (5.8, 5.2)'$

(5 marks)

(d) Let $\boldsymbol{X} = (X_1, X_2, \dots, X_p)'$ be a $p \times 1$ random vector and A be $q \times p$ a symmetric matrix of constants. Show that $E[\boldsymbol{X'AX}] = Trace(\boldsymbol{A}\Sigma) + \boldsymbol{\mu' A\, \mu}$ (5 marks)

(e) Two mixed bivariate normal populations P_1 and P_2 were discovered that the two populations can be separated. The population characteristics for the two populations were as follows

$P_1 \sim N_2(\mu_1, \Sigma)$ and $P_2 \sim N_2(\mu_2, \Sigma)$ where $\mu_1 = (6.2, 3.8)', \mu_2 = (5.8, 3.4)'$ and $\Sigma = \begin{bmatrix} 25 & 16 \\ 16 & 16 \end{bmatrix}$

Construct the optimal linear discriminant rule and classify the new observations given

$\underline{\overline{X}} = (6.0, 3.4)'$ (5 marks)

QUESTION TWO (20 MARKS)

(a) What is a discriminant analysis? (2 marks)

(b) In Antrophometic study, it is necessary to classify new observations into the population they belong to. Suppose that there are two populations P_1 and P_2 with probability distributions $f_1(x)$ and $f_2(x)$. Let R_1 and R_2 (R_1 and R_2 are such that $R_1 \cap R_2 \in \emptyset$) be the supports $f_1(x)$ and $f_2(x)$ respectively such that for all $\underline{X} \in R_1$, the distribution of $\underline{X}$ is $f_1(x)$ and for $\underline{X} \in R_2$, the distribution is $f_2(x)$. Let α_1 and α_2 denote the probability of classifying $\underline{X}$ into P_2 when it belongs to P_1 and the probability of classifying $\underline{X}$ into P_1 when it belongs to P_2 respectively. Suppose π_1 and π_2 be the probability that $\underline{X}$ arises from P_1 and P_2 respectively, construct a linear discriminant rule for classifying the new observations. (10 marks)

(c) An anthropologist wishes to classify a mixed sample of skulls believed to be from Homoerectus [P_1] and Homosapien [P_2] populations. The sample has a mean vector $\underline{\overline{X}} = \begin{bmatrix} \overline{X}_1 \\ \overline{X}_2 \end{bmatrix} = \begin{bmatrix} 6.2 \\ 4.0 \end{bmatrix}$ where $\overline{X}_1$ and $\overline{X}_2$ represent the jaw size mean and skull weight mean respectively. It is assumed that $P_1 \sim N_2(\mu_1, \Sigma)$ and $P_2 \sim N_2(\mu_2, \Sigma)$ where $\mu_1 = (6.0, 4.2)', \mu_2 = (5.5, 3.0)'$ and $\Sigma = \begin{bmatrix} 4 & 5 \\ 5 & 9 \end{bmatrix}$. Construct an optimal linear discriminant rule hence classify the new observations above(8 marks)

QUESTION THREE (20 MARKS)

(a) What is principal component analysis? (2 marks)

(b) In an experiment involving two correlated variables, the following sample statistics were obtained: $\underline{\overline{X}} = \begin{bmatrix} 10.00 \\ 10.00 \end{bmatrix}$ $S = \begin{bmatrix} 0.7986 & 0.6793 \\ 0.6793 & 0.7343 \end{bmatrix}$. Determine

(i) Principal components (8 marks)

(ii) Variance of each principal component (2 marks)

(iii) Percentage of variance explained by each principal component (3 marks)

(iv) Correlation of P.C's with original variables (5 marks)

QUESTION FOUR (20 MARKS)

Observations on three responses are collected from two treatments as shown in the table below

Response \ Treatment	1	1	1	1	2	2
X_1	7	8	9	6	7	5
X_2	12	15	13	10	12	10
X_3	6	6	7	5	7	5

Obtain

(i) Between Treatment sum of squares (6 marks)

(ii) Within treatment sum of squares (5 marks)

(iii)MANOVA table (2 marks)

(iv)Test at $\alpha = 0.05$level of significance that there is no treatment effect. (7 marks)

QUESTION FIVE (20 MARKS)

(a) Let $X_1 \sim N_2(\mu_1, \Sigma)$ and $X_2 \sim N_2(\mu_2, \Sigma)$. Independent random samples of size 10 and 9 were taken from X_1 and X_2 respectively. The summary statistics are as follows:

$$\underline{\bar{X}}_1 = \begin{bmatrix} 55 \\ 34 \end{bmatrix}, \underline{\bar{X}}_2 = \begin{bmatrix} 60 \\ 43 \end{bmatrix}, S_1 = \begin{bmatrix} 20 & 10 \\ 10 & 10 \end{bmatrix}, S_2 = \begin{bmatrix} 4 & 16 \\ 16 & 9 \end{bmatrix}$$

(i) Obtain the pooled sample variance-covariance matrix S_p (3 marks)

(ii) Test the hypothesis at $\alpha = 0.05$ level of significance

$$H_0: \mu = \mu_0$$
$$H_0: \mu = \mu_1$$

Where Σ is unknown (5 marks)

(b) For a bivariate normal distribution, use the data below to test at $\alpha = 0.05$ level the hypothesis

$$H_0: \mu = (3.4, 6)' \quad Vs$$
$$H_1: \mu = (3.4, 6)'$$

$$\underline{X} = \begin{bmatrix} 3 & 4 & 5 & 6 & 2 \\ 9 & 5 & 7 & 2 & 8 \end{bmatrix}$$ (7 marks)

(c) Find the symmetric matrix A for a quadratic form $Q(X_1, X_2, X_3) = 9X_1^2 + 16X_1X_2 + X_2^2 + 8X_1X_3 + 6X_2X_3 + 3X_3^2$. Hence obtain the expected value of $Q(X_1, X_2, X_3)$ given that $\mu = \begin{pmatrix} 6 \\ 7 \\ 3 \end{pmatrix}$ and $\Sigma = \begin{pmatrix} 1 & 2 & 5 \\ 2 & 4 & 3 \\ 5 & 3 & 9 \end{pmatrix}$ (5 marks)