MACHAKOS UNIVERSITY

University Examinations for 2022/2023 Academic Year

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

First YEAR FIRST SEMESTER EXAMINATIONS FOR

BACHELOR OF SCIENCE (MATHEMATICS AND STATISTICS)

BACHELOR OF SCIENCE (MATHEMATICS AND COMPUTER SCIENCE)

BACHELOR OF SCIENCE (STATISTICS AND PROGRAMMING)

BACHELOR OF EDUCATION

SMA 102: BASIC MATHS

DATE: **TIME:**

Instructions to Candidates.

Answer question ONE and any other TWO questions.

QUESTION ONE (COMPULSORY) (30 MARKS)

a) Express with real denominator the value of: $z = \frac{1}{5+4i} + \frac{1}{5-4i}$. (3 marks)

b) Show that if $A \subseteq B$ and $B \subseteq A$ then $A = B$ (3 marks)

c) Factorize $2x^3 - x^2 - 2x + 1$. Hence find the values of x for which $2x^3 - x^2 - 2x + 1 > 0$.

(4 marks)

d) Write $(5 + 3i)$ in the form $z = r(cos\theta + isin\theta)$. (5 marks**)**

e) All the students in an academy of 30 musicians play either the piano or the violin. In this academy 22 play the piano and 17 play the violin. Display this information in a Venn diagram hence find how many students play either the piano or violin but not both. (5 marks)

f) The coefficient of x in the expansion of $(1+ax)^5$ is equal to the coefficient of x^4 in the expansion of $\left(9+\frac{x}{3}\right)^6$, calculate the value of a . (5 marks)

g) Solve the equation $x^3 - 7x - 6 = 0$. (5 marks)

QUESTION TWO (20 MARKS)

a) How many arrangements can we have from the word $\boldsymbol{SUCCESS}$? (3 marks)

b) Solve the equation $x^4 + 5x^3 + 5x^2 - 5x - 6 = 0$ (7 marks)

c) Obtain the first five terms of the expansion $\sqrt{1 + 2x}$ in ascending powers of x, state the values of x for which the expansion is valid (5 marks)

d) Use induction proof that $1 + 2 + 3 + \cdots + n = \frac{1}{2}n(n + 1)$ (5 marks)

QUESTION THREE (20 MARKS)

a) Prove that $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$ (5 marks)

b) A survey was conducted on the first 41 presidents of the United States of America. The following information was gathered:

8 held cabinet posts,

14 served as vice-president,

15 served in the U.S.A Senate,

2 served in cabinet posts and as vice-president,

4 served in cabinet posts and in the U.S. Senate,

6 served in the U.S. Senate and as vice-president,

1 served in all three positions.

Required: Use a Venn diagram to determine how many presidents served in:

i. None of these three positions? (2 marks)

ii. Only in the U.S.A Senate. (2 marks)

iii. At least one of the three positions. (2 marks)

iv. Exactly two positions. (2 marks)

c) Expand $\sqrt{\frac{1+x}{1-x}}$ in ascending powers of x as far as the term in x^3 (7 marks)

QUESTION FOUR (20 MARKS)

a) State De Moivre’s Theorem. (1 mark)

b) Evaluate $\sqrt[4]{81(\cos 60^0 + i\, sin\, 60^0)}$ (8 marks)

c) Solve for x and y in the equation $\frac{2+5i}{1-i} = x + iy$. (5 marks)

d) Find the principal value of the argument of the complex number $z = \frac{1}{2} - \frac{\sqrt{3}}{2} i$. Hence write z in polar form, then use this expression to determine $z^{\frac{1}{3}}$ and z^4. (6 marks)

QUESTION FIVE (20 MARKS)

a) Show that $p \wedge (q \vee r)$ and $(p \wedge q) \vee (p \wedge r)$ are logically equivalent (4 marks)

b) A group of tourists is composed of six from Tokyo, seven from Moscow and eight from Nairobi.

i. In how many ways can a committee of six tourists be formed with two tourists from each city? (4 marks)

ii. In how many ways can a committee of seven tourists be formed with at least two tourists from each city? (4 marks)

c) Simplify completely $\left(\cos\frac{7}{3}\pi + i\sin\frac{7}{3}\pi\right)\left(\cos\frac{2}{3}\pi - i\sin\frac{2}{3}\pi\right)$ leaving your answer in cartesian/rectangular form. (4 marks)