MACHAKOS UNIVERSITY

University Examinations 2019/2020Academic year

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF PHYSICAL SCIENCES

FIRST YEAR FIRST SEMESTER EXAMINATION FOR

BACHELOR OF SCIENCE TELECOMMUNICATION ENGINEERING

BACHELOR OF SCIENCE IN MATHEMATICS

BACHELOR OF EDUCATION (SCIENCE)

BACHELOR OF SCIENCE IN MATHEMATICS

SPH 103: MATHEMATICS FOR PHYSICS

DATE: 11/12/2019 **TIME: 8.30-10.30 AM**

INSTRUCTIONS TO CANDIDATES

- The paper consists of **two** sections.
- Section **A** is **compulsory** (30 marks).
- Answer any **two** questions from section **B** (each 20 marks).

QUESTION ONE (30 MARKS) (COMPULSORY)

a) Find the probability that in a group of k-people at least two have the same birthday (ignoring 29th February). (3 marks)

b) The following set of data refers to the amount of money in £s taken by a newsvendor for 6 days. Determine the mean, median, and modal values of the set:

$\{27.90, 34.70, 54.40, 18.92, 47.60, 39.68\}$ (3 marks)

c) Find $f^{-1}(x)$ when $f(x) = 2x + 1, x \in \Re$ (3 marks)

d) Evaluate the limits; (3 marks)

$$\lim_{x\to 1}\left(x^2 + 2x^3\right),$$
$$\lim_{x\to 0}\left(x\cos x\right),$$
$$\lim_{x\to \pi/2}\left(\frac{\sin x}{x}\right)$$

e) Find the derivative with respect to x of: (4 marks)

$$x^3 - 3xy + y^3 = 2$$
$$y = a^x$$

f) Find the volume V of the parallelepiped with sides;

$\vec{A} = \hat{i} + 2\hat{j} + 3\hat{k}$, $\vec{B} = 4\hat{i} + 5\hat{j} + 6\hat{k}$ and $\vec{C} = 7\hat{i} + 8\hat{j} + 10\hat{k}$. (3 marks)

g) Given the two displacements; $\vec{D} = (6.0\hat{i} + 3.0\hat{j} - 1.0\hat{k})$ m; and $\vec{E} = (4.0\hat{i} - 5.0\hat{j} + 8.0\hat{k})$ m,

i. Find the magnitude of the displacement $2\vec{D} - \vec{E}$. (2 marks)

ii. Find the angle between the vectors. (2 marks)

h) Show that if $\vec{A} = \vec{B} + \lambda\vec{C}$, for some scalar λ, then $\vec{A} \times \vec{C} = \vec{B} \times \vec{C}$. (2 marks)

i) Solve the equation $x^2 + 8x + 10$ by completing the square method. (3 marks)

j) Find the points on the curve with equation $y = x^3 + 6x^2 + 5$ where the value of the gradient is -9. (3 marks)

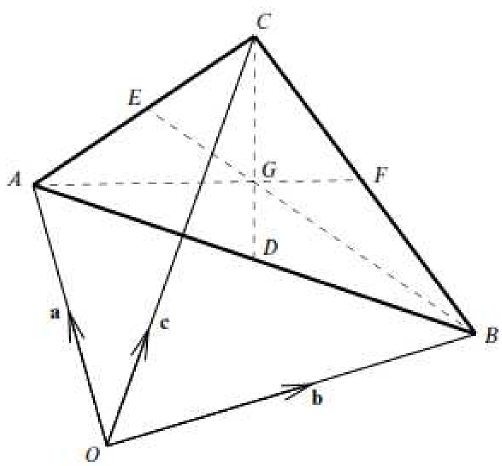
k) Evaluate $\int_0^2 \frac{dx}{4 + x^2}$. (3 marks)

l) Show that $\cosh 3x = 4\cosh^3 x - 3\cosh x$. (3 marks)

m) Find the roots of the cubic equation $x^3 + 4x^2 + x - 26 = 0$. (3 marks)

QUESTION TWO (20 MARKS)

a) The vertices of triangle **ABC** have position vectors ***a***, ***b*** and ***c*** relative to some origin 0 (see figure 1.0).



Find the position vector of the centroid G of the triangle. (5 marks)

b) Four non-coplanar points A,B,C,D are positioned such that the line AD is perpendicular to BC and BD is perpendicular to AC. Show that CD is perpendicular to AB. (5 marks)

c) With the help of an Argand diagram, distinguish between the modulus and argument of a complex number. (3 marks)

d) Find the modulus and argument of the complex number $-1+\sqrt{3i}$. Sketch the argand diagram. (4 marks)

e) Given that; $z_1 = 3+4i$ and $z_2 = 1-2i$, find (a) $z_1 + z_2$, (b) $z_1 - z_2$, and (c) $z_1 z_2$ (3 marks)

QUESTION THREE (20 MARKS)

a) A particle moving in the xy plane undergoes a displacement given by $\Delta\vec{r} = (2.0\hat{i} + 3.0\hat{j})m$ as a constant force $\vec{F} = (5.0\hat{i} + 2.0\hat{j})N$ acts on the particle. Calculate the work done by $\vec{F}$ on the particle. (3 marks)

b) (i) Find $\int (2x^3 - 3x + 4)dx$ (2 marks)

(ii) Evaluate $\int_0^{\frac{3}{2}} \frac{dx}{\sqrt{9-x^2}}$. (3 marks)

c) (i) Find the gradient of the scalar field; $f(x, y, z) = xy^2 - yz$ (3 mark)

(ii) Find the divergence of the vector field $F = 3x^2\hat{i} - 6xy\hat{j}$. Explain the result (2 marks)

(iii) Find the curl of the vector field $F = y^3\hat{i} + xy\hat{j} - z\hat{k}$. (3 marks)

d) Determine the probabilities of selecting at random (i) a man, and (ii) a woman from a crowd containing 20 men and 33 women. (4 marks)

QUESTION FOUR (20 MARKS)

a) (i) Write down the polar form of a complex number**.** (1 mark)

(ii) Find $z_1 z_2$ if $z_1 = 2\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)$ and $z_2 = 3\left(\cos\frac{\pi}{6} - i\sin\frac{\pi}{6}\right)$. (3 marks)

b) The functions $f, g,$ and h are defined by:

$$f(x) = 2x \text{ for } x \in \Re$$
$$g(x) = x^2 \text{ for } x \in \Re$$
$$h(x) = \frac{1}{x} \text{ for } x \in \Re, x \neq 0.$$

Find the following; (i) $fg(x)$ (ii) $gf(x)$ (iii) $gh(x)$ (iv) $f^2(x)$ (v) $fgh(x)$

(5 marks)

c) Find all the stationary points on the curve of $y = 2t^4 - t^2 + 1$ and sketch the curve.

(6 marks)

d) Find $(\sqrt{3} + i)^{10}$ in the form $a + ib$ (5 marks)

QUESTION FIVE (20 MARKS)

a) A force of $\vec{F} = (2.0\hat{i} + 3.0\hat{j})N$ is applied to an object that is pivoted about a fixed axis aligned along the z-coordinate axis. The force is applied at a point located at $\vec{r} = (4.0\hat{i} + 5.0\hat{j})m$. Find the torque $\vec{\tau}$ applied to the object. (4 marks)

b) Find $z_1 z_2$ if $z_1 = 2\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)$ and $z_2 = 3\left(\cos\frac{\pi}{6} - i\sin\frac{\pi}{6}\right)$. (4 marks)

c) The quartic equation $x^4 + 2x^3 + 14x + 15 = 0$ has one root equal to $1 + 2i$. find the other three roots (4 marks)

d) Write the mathematical expression for de Moivre's theorem. Using the theorem, simplify $\left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right)^3$. (3 marks)

e) (i) Find $f^{-1}(x)$ when $f(x) = x^2 + 2, x \geq 0$ (3 marks)

(ii) Find $f(7)$ and $f^{-1}f(7)$. What do you notice? (2 marks)