MACHAKOS UNIVERSITY

University Examinations 2018/2019

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS, STATISTICS AND ACTUARIAL SCIENCE

FOURTH YEAR SPECIAL/ SUPPLEMENTARY EXAMINATION FOR

BACHELOR OF SCIENCE IN STATISTICS AND PROGRAMMING

SMA 465: MEASURE AND PROBABILITY

DATE: 26/7/2019 **TIME: 11:00 – 1:00 PM**

INSTRUCTIONS:

Attempt Question one and any other two questions

QUESTION ONE (COMPULSORY) (30 MARKS)

a) Consider a non-empty set Ω. Define the following:

i) A measurable space,(Ω, Π)
ii) A measure on (Ω, Π), say , μ
iii) A measure μ whichis such that it is finite μ-a.e.

(12 marks)

b) State:

i) The Lebesgue Monotone Convergence Theorem.
ii) Fatou's Lemma (6 marks)

c) Prove any one of the results stated in Part (c). (4 marks)

d) Let P be a Probability measure. If **A** and **B** are some two events,

i) Define the conditional probability of**A** given **B**.
ii) Show that the conditional Probabilityof **A** given **B** is a Probability measure.

(8 marks)

QUESTION TWO (20MARKS)

a)

i) State the axioms of a measure, μ.

ii) Show that the function , m, defined on the theBorel algebra, B , by:

. m : $(\Re, B) \rightarrow (\Re, B)$.such thatm $\{(a_i,b_i]\} = b_i - a_i$, with a_i, b_i in $\Re$, $a_i \leq b_i$, for i=1,2,…. , is a measure. (10 marks)

b) Explain, for a given measure space(Ω, Π, μ),

i) Completion of the measure space

ii) μ -a.e

iii) the Lebesgue-Stieltjes measure space (10 marks)

QUESTION THREE. (20MARKS)

a) Show that for a non-empty set Ω:

i) The set $\{\Phi, \Omega\}$ is a σ-field

ii) The set $\{\Phi, A, A^c, \Omega\}$for $A \subset \Omega$,is aσ-field

(10 marks)

b) Show that if Π is a σ-field then $\bigcap_{i=1} A_i \in \Pi$ for all $A_i \in \Pi$, i = 1, 2, 3,_ (10 marks)

QUESTION FOUR (20 MARKS)

a) Consider a non-empty set Ω. Define the following:

i) A measurable space,(Ω, Π)

ii) A measure on (Ω, Π), say , μ

iii) A measure μ whichis such that it is finite μ-a.e. (12 marks)

b) Consider a function, **f**, which is given by:

$f : (\Omega_1, \Pi_1) \rightarrow (\Omega_2, \Pi_2)$

i) when is the function **f** said to be a measurable function? (4 marks)

ii) Suppose$\Omega_1 = \Omega_2 = \Re U \{-\infty, \infty\}$ and$\Pi_1 = \Pi_2 =$ Borel opensubsetsin $\Re U\{-\infty, \infty\}$. Show that the constant function , for some c in $\Re$;f(x)=c, is a measurable function. (8 marks)

QUESTION FIVE. (20 MARKS)

a) State the Chebychev's inequality. (5 marks)

b) Prove the Chebychev's inequality. (5 marks)

c) Let X_n: $n \geq 1$ be a sequence of random variables with $E(X_n) = \boldsymbol{\mu}$,

for all X_n: $n \geq 1$. Show that the sample mean $\bar{X}_n$ converges to $\boldsymbol{\mu}$ in probability.

(10 marks)