MACHAKOS UNIVERSITY

University Examinations 2020/2021 Academic Year

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

SMA 102 : BASIC MATHEMATICS

ANSWER QUESTIONS ONE AND ANY OTHER TWO.

Question one (COMPULSORY) (30 MARKS)

a. Distinguish the following terms in relation to propositions
   - i) Disjunction (2marks)
   - ii) Conjunction (2 marks)
   - iii) Negation (2 marks)
   - iv) Tautology (2 marks)
   - v) Contradiction (2 marks)

b. Given that $A=\{1,2,3,4,5,6\}$ and $B=\{2,4,6,7\}$, find
   - i) $A-B$ (2 marks)
   - ii) $B-A$ (2 marks)
   - iii) $A\Delta B$ (2 marks)

c. State the following theorems
   - i) De moivres theorem (2 marks)
   - ii) Remainder theorem (2 marks)
   - iii) Factor theorem (2 marks)
   - iv) Binomial theorem (2 marks)

d. Determine the coefficient of the fourth and seventh terms in the expansion for $(1+2x)^{100}$ (4 marks)

e. Find the modulus of the complex number $z=4i$ (2marks)

Question two (20 marks)

a. Prove that $\sqrt{2}$ is irrational (4 marks)

b. In how many ways can 4 boys and 2 girls be seated in a row if the girls must not be separated? (3 marks)

c. Express the following complex number $z=2\left(\cos\frac{\pi}{6}+i\sin\frac{\pi}{6}\right)$ in Cartesian form. (3 marks)

d. A polynomial has a remainder 9 when divided by $x-3$ and a remainder -5 when divided by $2x+1$. Find the remainder when $f(x)$ is divided by $(x-3)(2x+1)$

(5marks)

e. Find the value without using a calculator

i) ${}^{9}p_{3}$ (2marks)

ii) ${}^{9}C_{3}$ (2marks)

iii) ${}^{9}P_{3}\div{}^{9}C_{3}$ (1mark)

Question three (20marks)

a. Out of 60 students who exercise regularly, 25 jog, 20 play basketball and 15 swim, 10 play basketball and jog, 5 play basketball and swim, 7 jog and swim and 3 people do all the three. How many students don't do any of the these activities? (6 marks)

b. Given that $z=2-5i$, represent it in a well labelled Argand diagram. (4 marks)

c. The function $f(x)=ax^3+bx^2+7x+6$ has a remainder 6 if divided by $x-1$, and is exactly divisible by $x-2$. Find the values of a and b and hence factorise $f(x)$ completely. (6 marks)

d. Show using a truth table that $p\vee\sim(p\wedge q)$ is a tautology (4 marks)

Question four (20marks)

a. In how many ways can four boys and two girls be seated in a row if

i) the girls must not be separated. (2marks)

ii) the girls are separated. (2marks)

b. In how many ways can a 7 handball players be selected from a team of 12 players to participate in a match. (2 marks)

c. In how many ways can a class of 20 students be divided, if the class contains one set of twins who must not be separated? (2marks)

d. Apply the remainder theorem to determine the remainder in $(5x^3 - 4x^2 - 3x + 6) \div (x - 2)$ (3 marks)

e. Using long division, find the remainder in $(x^3 + 3x^2 - 13x - 10) \div (x + 5)$ (3marks)

f. Consider the propositions

P: Mathematicians are generous

Q: spiders hate algebra

Write the compound propositions symbolized by

i) $\sim (P \wedge Q)$

ii) $\sim P \leftrightarrow Q$ (6 marks)

<u>Question five (20marks)</u>

a. Construct the truth table for the following proposition and determine whether it is either a tautology, contradiction, or contingency.

$[(p \rightarrow q) \vee \sim q] \rightarrow \sim p$ (4 marks)

b. Show that $[(p \rightarrow q) \wedge \sim q] \Rightarrow \sim p$ (3 marks)

c. Show that $\sim(p \vee q) \equiv \sim p \wedge \sim q$ (3 marks)

d. Find the value of c and d if $cx^4 + dx^3 - 8x^2 + 6$ has a remainder $2x+1$ when divided by $x^2 - 1$. (5 marks)

e. Express $\left(\frac{1+3\sqrt{2}i}{1-i}\right)^2$ in polar form (5 marks)