MACHAKOS UNIVERSITY

University Examinations 2016/2017

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

FOURTH YEAR FIRST SEMESTER EXAMINATION FOR

BACHELOR SCIENCE (MATHEMATICS AND COMPUTER SCIENCE)

BACHELOR SCIENCE (STATISTICS AND PROGRAMMING

BACHELOR SCIENCE (MATHEMATICS)

BACHELOR EDUCATION (SCIENCE)

SMA 463: TIME SERIES ANLYSIS

DATE: 28/7/2017 **TIME: 2:00 – 4:00 PM**

INSTRUCTIONS

Answer question ONE (Compulsory) and any other TWO questions

QUESTION ONE: (COMPULSORY) (30 MARKS)

a) i Describe the **FOUR** components of time series (8 marks)

ii Explain the main objective of a time series analysis. (2 marks)

b) Explain the following statements

i Time series is stationary in the weak sense. (3 marks)

ii Time series is stationary in the strict sense. (3 marks)

c) State and explain the TWO main models of time series analysis (4 marks)

d) Given that $X_t = e^{it\theta}$,show that the effect of the first difference filter, the series is magnified by $g(\theta) = 2sin\frac{\theta}{2}$ (5 marks)

e) Let $X_t: t = 0, \mp 1, \mp 2, \ldots\ldots$ be a stochastic process given by $X_t = a + bt + e_t$ where e_t is a sequence of independent random variables distributed with $\mu = 0, and\ \sigma^2$ as $e_t \sim N(0, \delta^2)$.Show that $X_t = (X_t - X_{t-1})$ is stationary (5 marks)

QUESTION TWO (20 MARKS)

a) Given that $Y_t = Sin\theta t$, Show that it is a weak series and determine the covariance function of the series (8 marks)

b) Define a moving average process of order q [MA(q)] hence:

   i. Determine its mean and variance.

   ii. Obtain its covariance and autocorrelation functions.

   iii. State whether it is a second order stationary. (12 marks)

QUESTION THREE (20 MARKS)

A polynomial of degree p = 2 is fitted to m = 2k + 1, consecutive points of a time series. Show that the trend value corresponding to the middle point of the set can be obtained as a weighed average of the observed value. (9 marks)

Further

a) Determine $[m, 2]$ in term of weights Cj's (1 mark)

   and show that

b) The weights are symmetric about their mid-value (3 marks)

   and

c) The sum of weights in $[m, 2]$ is unity (3 marks)

d) Determine [5,2] (4 marks)

QUESTION FOUR (20 MARKS)

a) Consider two MA(1) processes P and Q given by

$$P: X_t = e_t + \lambda e_{t-1}$$

$$Q: X_t = e_t + \frac{1}{\lambda} e_{t-1}$$

Obtain the autocorrelation function for the two processes. Is a moving average process uniquely identified from a given autocorrelation function give reason. (5 marks)

b) Consider the autogressive process of order 2 given by

$X_t = \propto_1 x_{t-1} + \propto_2 x_{t-2} + e_t$ where

(e_t) is a purely random process.

i. Derive the Yule-Walker equation and hence its general solution. What are the values of $\propto_1$ and $\propto_2$ for which the process is stationary. (8 marks)

ii. If $\propto_1 = \frac{1}{3}$ and $\propto_2 = \frac{2}{9}$ show that the autocorrelation function of X_t is given by.

$\rho(h) = \frac{16}{2}\,(\frac{2}{3})^h + \frac{5}{21}\,(\frac{-1}{3})^h$, h = 0, 1, 2. (7 marks)

QUESTION FIVE (20 MARKS)

a) Let $X_t = x_{t-1} + e_t$ where the sequence e_t is a sequence of uncorrelated random variable with mean μ and variance δ^2 and $x_0 = 0$,Show that the time series in non Stationary

(10 marks)

b) Consider an AR(1) given by $\alpha X_t + e_t$, where e_t is a purely random process and $|\alpha| < 1$ and let $f^*(\lambda)$ be the normalized spectrum of X_t

Show that $f^*(\lambda) = \frac{1-\alpha^2}{2\pi(1-2\alpha cos\lambda+\alpha^2)}$ (10 marks)