



MACHAKOS UNIVERSITY

ISO 9001:2008 CERTIFIED

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

FOR THE BACHELOR'S DEGREE

UNIVERSITY SUPPLEMENTARY EXAMINATIONS

SMA 104: CALCULUS I

Time 2hours

Attempt question *one (compulsory)* and any other *two questions*.

Question 1(30marks)

a.) Determine the inverse of the function $f(x) = \sqrt{4-x}$ (3marks)

b.) Evaluate

$$\lim_{x \rightarrow 2} \frac{x-2}{x^2-x-2} \quad (3marks)$$

c.) Consider the function $h(x)$ such that $h: x \rightarrow x^2$ defined on the set $\mathfrak{R}$ by the elements

$x_1 = -2$, $x_2 = 2$. Show that $h(x)$ is a surjective function. (3marks)

d.) Prove the continuity of the function $f(x) = 3x + 1$ at the point $x = 1$ (3marks)

e.) Approximate $\sin 29^\circ$ using small changes given that $1^\circ = 0.0175^c$ (3marks)

f.) Calculate $\frac{d^2 y}{dx^2}$ for the parametric equations $x = 2t$, $y = t^2 - 1$ (3marks)

g.) Evaluate $\frac{dy}{dx}$ for;

i.) $y = \frac{\sqrt{\cos x}}{x^2 \sin x}$ (4marks)

ii.) $f(x) = x^2 y + 4y = 8$ at the point $(2,1)$ (4marks)

iii.) $f(x) = (12x^3 - 4x^2 - x + 1)^4$ (4marks)

Question 2(20marks)

Consider the function $f(x) = x^5 - 5x^3$

- a) Determine using the second derivative
 - i.) Local maximum point (3marks)
 - ii.) Local minimum point (3marks)
- b) Work out the coordinate of the points of inflection of $f(x)$ (4marks)
- c) Discuss the concavity of $f(x)$ (5marks)
- d) Sketch the graph of $f(x)$ (5marks)

Question 3(20marks)

- a.) Given that $x = \sin t$, $y = 5 \cos t$ determine $\frac{dy}{dx}$ when $t = \frac{\pi}{3}$ (4marks)
- b.) Determine $\frac{dy}{dx}$
 - iii.) $y = \ln \left[\frac{x\sqrt{x+5}}{(x-1)^3} \right]$ (4marks)
 - iv.) $y = \sec^3 x$ (4marks)
 - v.) $y = x^{x^{x^\infty}}$ (4marks)
 - vi.) $y = \sinh^{-1} x$ (4marks)

Question 4 (20marks)

- a.) A closed rectangle box is made of very thin sheet metal and its length is three times its width. If the volume of the box is 288cm^3 ;
 - i) Show that its surface area is equal to $s = \frac{768}{x} + 6x^2$ where $x \text{ cm}$ is the width of the box. (2marks)
 - ii) Determine the dimensions of the box of least surface area (2marks)
 - iii) State the value of this surface area. (2marks)
- b.) Given that $y = 4^{3x} - 5e^{-3x}$ show that $\frac{d^2y}{dx^2} = 9y$ (4marks)

c.) Given that $f(x) = (\cos x - 2^x)(\ln x + 3 \sin x)$, use the product rule to obtain $f'(x)$ (5marks)

d.) Given that $f(x) = \frac{ax+b}{cx+d}$, use the quotient rule to show that $f'(x) = \frac{\begin{vmatrix} a & b \\ c & d \end{vmatrix}}{(cx+d)^2}$ (5marks)

Question 5(20marks)

a.) Given two functions: $f(x) = 3x - 2$, $g(x) = 8 - 9x$, determine;

i) $g[f(x)]$ (3marks)

ii) $f^{-1}(x)$ (3marks)

b.) Consider the Folium of Descartes equation ; $x^3 + y^3 - 6xy = 0$

i) Determine its tangent slope at the point $\left(\frac{4}{3}, \frac{8}{3}\right)$ (3marks)

ii) Calculate its normal line equation at the point $\left(\frac{4}{3}, \frac{8}{3}\right)$ (3marks)

c.) A particle is moving in a straight line and its distance s metres from a fixed point in the line after t seconds is given by the equation $s = 12t - 15t^2 + 4t^3$. Determine:

i) The velocity of the particle after 3 seconds (2marks)

ii) Acceleration of the particle after 3 seconds (2marks)

iii) Distance travelled between the two times when the velocity is instantaneously zero. (4marks)