



MACHAKOS UNIVERSITY

University Examinations 2019/2020 Academic Year

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

.....YEAR SEMESTER SPECIAL /SUPPLEMENTARY EXAMINATION FOR

BACHELOR OF SCIENCE IN

SMA 404: COMPLEX ANALYSIS II

DATE:

TIME:

INSTRUCTIONS: ANSWER QUESTION ONE AND ANY OTHER TWO QUESTIONS

QUESTION ONE (30 MARKS)

- a) State without proof the Residue theorem (2 marks)
- b) Determine all the poles of the function $f(z) = \frac{z+2}{z^2(z^2+9)}$ stating the order of each pole. (4 marks)
- c) Evaluate the integral $\int_c \frac{5z-3}{z(z-3)} dz$, $|z|=3$ using the Residue Theorem (5 marks)
- d) Find the transformation that maps the cycloid whose parametric equations are $x = a(t - \sin t)$, $y = a(1 - \cos t)$ into a straight line (6 marks)
- e) Investigate the convergence of the product $\prod_{n=2}^{\infty} (1 - \frac{1}{n^2})$ (6 marks)
- f) Prove that $f_2(z) = \frac{1}{1+i} \sum_{n=0}^{\infty} \left(\frac{z+i}{1+i} \right)^n$ is an analytic continuation of $f_1(z) = \sum_{n=0}^{\infty} z^n$, showing graphically the regions of convergence of the series. (7 marks)

QUESTION TWO (20 MARKS)

- a) State the Schwarz reflection principle (2 marks)
- b) Determine whether the following functions satisfies the Schwarz- reflection principle
- i) $f(z) = z^2 + 1$ (3 marks)
- ii) $f(z) = z^2 + i$ (3 marks)
- c) Prove that for closed polygons, the sum of the exponents $\frac{\alpha_1}{\pi} - 1, \frac{\alpha_2}{\pi} - 1, \dots, \frac{\alpha_{n-1}}{\pi} - 1$ in the Schwarz-Christoffel transformation is equal to -2 . (5 marks)
- d) Find the residues of $f(z) = \frac{z^2 - 2z}{(z+1)^2(z^2 + 4)}$ at all its poles in the finite plane (7 marks)

QUESTION THREE (20 MARKS)

- a) Differentiate between conformal mapping and isogonal mapping (3 marks)
- b) Consider the function $u(x, y) = x^3 - 3xy^2$
- i. Show that $u(x, y)$ is harmonic (2 marks)
- ii. Determine the conjugate harmonic of $u(x, y)$ (5 marks)
- c) Consider the lines $l_1 : x = 2$ and $l_2 : y = 2$ meeting at $z = z_o(2, 2)$. Discuss the co formality of $T(z) = z^2$ at z_o . (5 marks)
- d) Show that $f_1(z) = \sum_{n=0}^{\infty} z^n$ and $f_2(z) = \frac{1}{1-z}$ are analytic continuation of each other (5 marks)

QUESTION FOUR (20 MARKS)

- a) Investigate the convergence of the product $\prod_{n=2}^{\infty} (1 - \frac{1}{n^2})$ (5 marks)
- b) Show that $f_1(z) = \sum_{n=0}^{\infty} z^n$ and $f_2(z) = \frac{1}{1-z}$ are analytic continuation of each other (5 marks)
- c) Consider the triangle A(0,1), B(1,1), C(1,0) on the z-plane with image under the transformation $T(z) = z^2$. Show that the transformation T(z) maintains the angle at A,B and C respectively. (10 marks)

QUESTION FIVE (20 MARKS)

- a) Prove that the transformation of $u(x, y) = x^2 - y^2 + 2y$ is also harmonic under the transformation $z = w^3$ (10 marks)
- b) Show that the mapping $f(z) = z^2$ is conformal at the point $z = 1 + i$ with $C_1: x = 1$ and $C_2: y = i$. Find the angle of rotation and sketch the curves. (10 marks)