



MACHAKOS UNIVERSITY

University Examinations 2022/2023 Academic Year

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

THIRD YEAR SPECIAL/SUPPLEMENTARY EXAMINATION

SCO 108: DISCRETE MATHEMATICS

DATE: 6/3/2023

TIME: 8.30-10.30 A.M

Instructions to the Candidate:

1. This paper consists of **Five** questions.
2. Answer **Question 1** and any other **Two** questions.
3. You **must** have a scientific calculator for this paper.

SECTION A

QUESTION ONE 30 Marks (Compulsory)

- a) Define the following terms.
- i). A graph
 - ii). trail
 - iii). Set
 - iv). Proposition
 - v). A subset (5 marks)
- b) Find the power set of the set $A = \{1, 2, 3, 4\}$. (3 marks)
- c) Construct all the unlabeled graphs with 4 vertices (5 marks)
- d) Construct the truth table for the disjunction of two propositions (4 marks)

- e) In how many distinguishable ways can the product $Z^7 X^8 Y^7 T^6$ be arranged without using exponents. (3 marks)
- f) Given $A = ((abc)'c)'(a' + c)(b' + ac)'$ express it as a sum of product expression. (5 marks)
- g) Prove that if a bipartite graph has a cycle then all its cycles are of even length. (5 marks)

QUESTION TWO 20 MARKS

- a) Construct the logic circuit for the following output $Y = (XY + ZY')' + (X' + ZY)'$ (5 marks)
- b) Show that D_{210} (where D_{210} are divisors of 210) is a Boolean algebra
- Find the atoms (3 marks)
 - Find the subalgebra (3 marks)
 - Construct the lattice diagram (4 marks)
- c) Given that $X = 00111001$ $Y = 11100011$ $Z = 00110010$ $T = 01011011$. Find
- $$A = X.Y.Z.T + TZ \quad (5 \text{ marks})$$

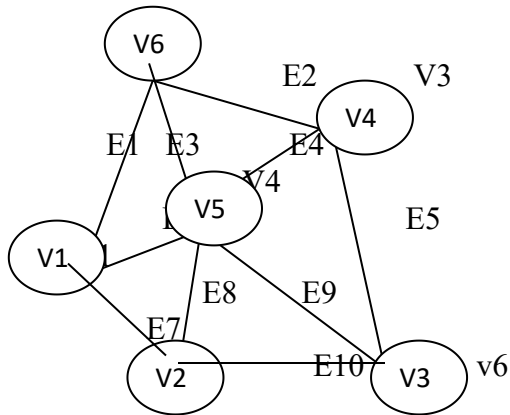
QUESTION THREE 20 MARKS

- a) Given that a and b are rational with $b \neq 0$ and s is an irrational number such that $a - bs = t$. Show that t is irrational hence show that $\frac{-1+\sqrt{3}}{1+\sqrt{3}}$ is irrational (6 marks)
- b) Show that $\sqrt{5}$ is irrational (5 marks)
- c) Proof that set of all even natural numbers is countable. (5 marks)
- d) Suppose two boys say Fred and Sum are playing a Football tournament such that the first person to win two games in a row or who wins a total of three games wins the tournament. Construct a rooted tree to illustrate the above. Find the number of ways the tournament can be won. (5 marks)

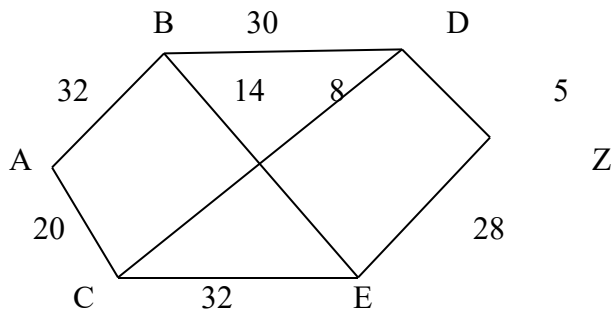
QUESTION FOUR 20 MARKS

- a) Prove that if G is a connected planar graph with P vertices and q edges. Where $p \geq 3$. then $q \leq 3p - 6$. (5 marks)

- b) Construct the Incidency matrix for the following graph. (5 marks)



- c) Find the shortest path from A to Z for the map shown below (5 marks)



- d) Prove that if M is a map with V vertices, E edges and R regions and K components. Then
 $V - E + R = K + 1$. (5 marks)

QUESTION FIVE 20 MARKS

- a) Let $U = \{i, j, k, l, m, n, o, p, q, r, s, t, u\}$, $A = \{i, k, l, m, q\}$ $B = \{j, k, q, r\}$
 $C = \{j, k, m, o\}$ and $D = \{j, o, p\}$.

Determine the set

- $A \cup B$
- $A \cap C$
- $(A \cup B) \cap C^c$
- $(C \cap A) \cup D$
-

(8 marks)

- b) Let $A = \{s, t\}$ and $B = \{1,4,6\}$ determine the set
 $(A \times B) \times B$ (2marks)
- c) A man who works five days a week can travel to work on foot, by bicycle or by bus. In
how many ways can he arrange a week's travelling to work? (6 marks)
- d) Show that $p \vee q$ and $p \rightarrow q$ are logically equivalent (6 marks)