MACHAKOS UNIVERSITY

University Examinations 2020/2021 Academic Year

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

FIRST YEAR SPECIAL/SUPPLEMENTARY EXAMINATION FOR

BACHELOR OF SCIENCE (INFORMATION TECHNOLOGY)

BACHELOR OF SCIENCE (COMPUTER SCIENCE)

SCO 111/SIT192: DIFFERENTIAL CALCULUS FOR COMPUTER SCIENCE

DATE: 11/8/2021 **TIME: 2.00-4.00 PM**

INSTRUCTIONS:

*Answer Question **ONE** which is compulsory and any other **TWO** Questions*

QUESTION ONE (30 MARKS)

a) Determine the $\lim\limits_{x\to\infty}\dfrac{5x^2-3x+2}{10x^2-x+100}$ (4 marks)

b) State the $L'H\hat{o}pital's\ Rule$, hence or otherwise evaluate

$$\lim_{x\to -3}\frac{x+3}{x^2+5x+6}$$

(3 marks)

c) Determine $\frac{dy}{dx}$ given that $y=\dfrac{\sin x}{x}$ (3 marks)

d) Find the derivative of the following function from the first principles.

$f(x)=x^2$ (4 marks)

e) If $y=\dfrac{u^2-1}{u^2+1}$ and $u=\sqrt[3]{x^2+2}$ find $\dfrac{dy}{dx}$, (5 marks)

f) Given that $f(0)=8,\ g(0)=5,\ f'(0)=3, g'(0)=1$, Find $F'(0)$ where $F(x)=\dfrac{f(x)}{g(x)}+3x^2+4$

(3 marks)

g) Determine if the following function is continuous. $g(x)=\begin{cases}+1 if x\geq 0\\ -1 if x<0\end{cases}$ (4 marks)

h) Find the derivative the following functions $y=x^2-4x$ and $x=\sqrt{2t^2+1},$ at $t=\sqrt{2}$ using chain rule , (4 marks)

QUESTION TWO (20 MARKS)

a) Using logarithmic differentiation and differentiate the following functions;

$xe^x \sin x$ (3 marks)

$te^t \cos t$

b) Find the gradient of the curve $x=\dfrac{t}{1+t}$, and $y=\dfrac{t^3}{1+t}$ at the point $\left(\dfrac{1}{2},\dfrac{1}{2}\right)$ (4 marks)

c) Determine $\dfrac{dy}{dx}$ given that $x^2+2xy-2y^2+x=2$, at the point $(-4,1)$ (4 marks)

d) Write down the derivative of $\ln\left(\dfrac{1}{\sqrt{x}}\right)$ (4 marks)

e) Differentiate $y=3x^2+\cos 2x$ from the first principles. (5 marks)

QUESTION THREE (20 MARKS)

a) Evaluate $\lim\limits_{x\to 0}\dfrac{\sqrt{2+x}-\sqrt{2}}{x}$ (3 marks)

b) Obtain $\dfrac{df(x)}{dx}$ for $f(x)=\dfrac{\sin x+e^{2x}}{\sin x}$ (4 marks)

c) Ink is dropped onto a blotting paper forming a circular stain which increases at the rate of $5cm^2/s$. Find the rate of change of the radius when the area is $30cm^2$ (6 marks)

d) Evaluate the derivative of the function given as $x^{\cos x}$ with respect to x, with respect to x, (7 marks)

QUESTION FOUR (20 MARKS)

a) If $x = t^3, y = t^2 - t$ find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ (6 marks)

b) Differentiate $y = \left(x^2+1\right)^3\left(x^3+1\right)^2$ (5 marks)

c) The curve of the function $f(x) = \alpha x^5 + \beta x^4 + 5x^3 - 1$ passes through (1,0) and has a stationary point at (1,0). Find the value of α, β , the other turning points and hence sketch the curve

(9 marks)

QUESTION FIVE (20 MARKS)

a) Find $\frac{dy}{dx}$ given that $x^2 + 2xy - 2y^2 + x = 2$ at the point $(-4,1)$ (6 marks)

b) Determine the dimensions that would minimize the total surface area of an open rectangular container if it is to have a volume of $32m^3$ (8 marks)

c) Determine the domain for each of the following functions

i. $y = x^3 + 3x - 6$ (1 mark)

ii. $y = \frac{1}{x^2 + 6x + 9}$ (2 marks)

d) Differentiate $x = t^3 + t^2, y = t^2 + t$ (3 marks)