MACHAKOS UNIVERSITY

University Examination 2018/2019

SCHOOL OF ENGINEERING AND TECHNOLOGY

DEPARTMENT OF ELECTRICAL AND ELECTRONICS ENGINEERING

FIFTH YEAR FIRST SEMESTER EXAMINATION FOR

BACHELOR OF SCIENCE (TELECOMMUNICATION AND IT)

EEE 546: INFORMATION THEORY AND CODING

DATE: 16/5/2019 **TIME: 8.30-10.30 AM**

INSTRUCTIONS

Answer Question ONE and any other TWO Questions.

QUESTION ONE (30 MARKS)

a) Draw the block diagram of a digital communication system that includes all the core elements and explain the function of each block. (12 marks)

b) The external operation of the digital communication system can be described using which three core parameters? (3 marks)

c) From an information theory point of view, what is entropy? (3 marks)

d) For a binary random variable X , $W_X = \{0, 1\}$, $p_X(1) = q$. Calculate its entropy and show how it varies with the value of q. (4 marks)

e) A communication channel has a bandwidth of roughly 100MHz. We would like to transmit information at a bit rate of 500Mbps. Is a signal-to-noise ratio of 30dB enough to reliably transmit this much information? Why or why not?. (4 marks)

f) We are given a medium that will reliably transmit frequencies between 0 and 25,000Hz. Is it possible to transmit 200Kbps of information along this medium? If so, then describe a method and any conditions that must be satisfied. If not, explain why. (4 marks)

QUESTION TWO (25 MARKS)

Let X and Y represent random variables with associated probability distributions p(x) and p(y), respectively. They are not independent. Their conditional probability distributions are p(x|y) and p(y|x), and their joint probability distribution is p(x, y).

a) What is the marginal entropy H(X) of variable X, and what is the mutual information of X with itself? (3 marks)

b) In terms of the probability distributions, what are the conditional entropies H(X|Y) and H(Y |X)? (3 marks)

c) What is the joint entropy H(X, Y), and what would it be if the random variables X and Y were independent? (3 marks)

d) Give an alternative expression for H(Y) − H(Y |X) in terms of the joint entropy and both marginal entropies. (2 marks)

e) What is the mutual information I(X; Y)? (3 marks)

f) What is the entropy H, in bits, of the following source alphabet whose letters have the probabilities shown? (2 marks)

A	B	C	D
1/4	1/8	1/2	1/8

g) Why fixed length codes inefficient for alphabets are whose letters are not equiprobable? Discuss this in relation to Morse Code. (2 marks)

h) Offer an example of a uniquely decodable prefix code for the above alphabet which is optimally efficient. What features make it a uniquely decodable prefix code? (2 marks)

QUESTION THREE (20 MARKS)

a) Calculate the probability that if somebody is "tall" (meaning taller than 6 ft or whatever), that person must be male. Assume that the probability of being male is p(M) = 0.5 and so likewise for being female p(F) = 0.5. Suppose that 20% of males are T (i.e. tall): p(T|M) = 0.2; and that 6% of females are tall: p(T|F) = 0.06. So this exercise asks you to calculate p(M|T).

If you know that somebody is male, how much information do you gain (in bits) by learning that he is also tall? How much do you gain by learning that a female is tall? Finally, how much information do you gain from learning that a tall person is female? (5 marks)

b) The input source to a noisy communication channel is a random variable X over the four symbols a, b, c, d. The output from this channel is a random variable Y over these same four symbols. The joint distribution of these two random variables is as follows:

	$x=a$	$x=b$	$x=c$	$x=d$
$y=a$	$\frac{1}{8}$	$\frac{1}{16}$	$\frac{1}{16}$	$\frac{1}{4}$
$y=b$	$\frac{1}{16}$	$\frac{1}{8}$	$\frac{1}{16}$	0
$y=c$	$\frac{1}{32}$	$\frac{1}{32}$	$\frac{1}{16}$	0
$y=d$	$\frac{1}{32}$	$\frac{1}{32}$	$\frac{1}{16}$	0

i. Write down the marginal distribution for X and compute the marginal entropy H(X) in bits. (2 marks)

ii. Write down the marginal distribution for Y and compute the marginal entropy H(Y) in bits. (3 marks)

iii. What is the joint entropy H(X, Y) of the two random variables in bits? (3 marks)

iv. What is the conditional entropy $H(Y|X)$ in bits? (3 marks)

v. What is the mutual information $I(X; Y)$ between the two random variables in bits? (2 marks)

vi. Provide a lower bound estimate of the channel capacity C for this channel in bits. (2 marks)

QUESTION FOUR (20 MARKS)

a) Consider Shannon's third theorem, the Channel Capacity Theorem, for a continuous communication channel having bandwidth W Hertz, perturbed by additive white Gaussian noise of power spectral density N_0, and average transmitted power P.

i. Is there any limit to the capacity of such a channel if you increase its signal-to-noise ratio P/N_0W without limit? If so, what is that limit? (2 marks)

ii. Is there any limit to the capacity of such a channel if you can increase its bandwidth W in Hertz without limit, but while not changing N_0 or P? If so, what is that limit? (2 marks)

b) What class of continuous signals has the greatest possible entropy for a given variance (or power level)? What probability density function describes the excursions taken by such signals from their mean value? (2 marks)

c) What does the Fourier power spectrum of this class of signals look like? How would you describe the entropy of this distribution of spectral energy? (2 marks)

d) An error-correcting Hamming code uses a 7 bit block size in order to guarantee the detection, and hence the correction, of any single bit error in a 7 bit block. How many bits are used for error correction, and how many bits for useful data? If the probability of a single bit error within a block of 7 bits is $p = 0.001$, what is the probability of an error correction failure, and what event would cause this? (2 marks)

e) Suppose that a continuous communication channel of bandwidth W Hertz and a high signal-to-noise ratio, which is perturbed by additive white Gaussian noise of constant power spectral density, has a channel capacity of C bits per second. Approximately how much would C be degraded if suddenly the added noise power became 8 times greater? (2 marks)

f) You are comparing different image compression schemes for images of natural scenes. Such images have strong statistical correlations among neighbouring pixels because of the properties of natural objects. In an efficient compression scheme, would you expect to find strong correlations in the compressed image code? What statistical measure of the code for a compressed image determines the amount of compression it achieves, and in what way is this statistic related to the compression factor? (3 marks)

g) Define what is meant by:

   i. Binary frequency shift keying (binary FSK)

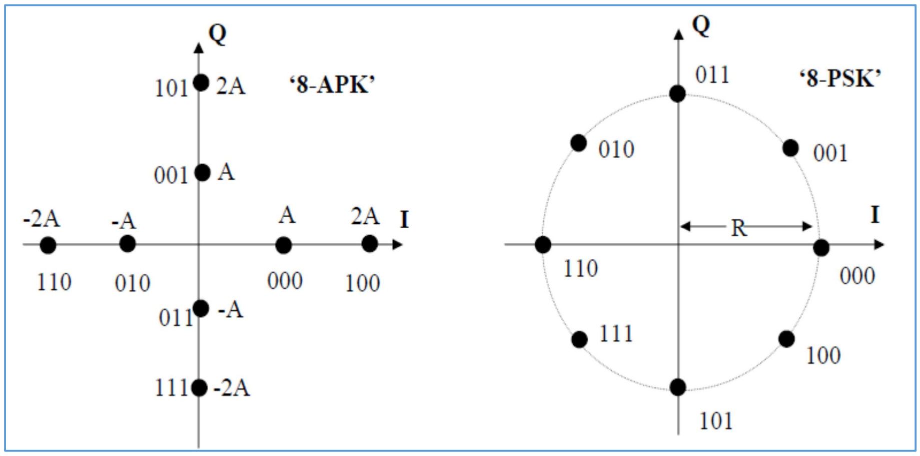
   ii. Gaussian minimum shift keying (GMSK)

   What are the main advantages of GMSK that led to its adoption for second generation GSM mobile telephones? Show how GMSK may be generated. (5 marks)

QUESTION FIVE (20 MARKS)

a) Why is Gray coding used in multi-level digital modulation schemes? (2 marks)

b) The constellation diagrams for an '8-APK' modulator and an '8-PSK' modulator are shown below:



Assuming all symbols are equally likely, calculate the average energy per bit, E_b, for each of these modulation schemes when the symbol rate is 1 kbaud. Show that if $R \approx 1.58A$, the average energy per bit is the same for both schemes. (5 marks)

Symbol detection is achieved by choosing the nearest of the eight symbols on the constellation diagram to a received symbol and therefore the minimum distance between symbols is taken as a measure of noise immunity. Which of the two schemes has the greater noise immunity for a given E_b/N_0? (3 marks)

Sketch the waveform produced by each of the schemes above when the 3 kb/s bit-stream is as follows:

0 0 0 1 1 0 0 1 1 1 0 1

and the carrier frequency is 2 kHz. The effect of band-limiting and pulse-shaping need not be shown. (5 marks)

c) Consider a (6,3) linear block-code that is generated by a generator matrix:

$$\mathbf{G} = \begin{bmatrix} 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 & 1 \end{bmatrix}$$

Is the code systematic? Write down the check-bit equations and tabulate the code words and their weights. Find the parity check matrix **H** for this code. What is the minimum distance? How many errors can be:

i. corrected
ii. detected

with this code in hard decoding? How would you do the actual error correction / detection? (5 marks)