MACHAKOS UNIVERSITY

University Examinations 2016/2017

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

FIRST YEAR SECOND SEMESTER SUPPLEMENTARY/SPECIAL EXAMINATION FOR
BACHELOR OF ELECTRICAL AND ELECTRONICS ENGINEERING
MECHANICAL ENGINEERING
CIVIL AND BIULDING ENGINEERING

ECU 107: ENGINEERING MATHEMATICS IV

DATE: **TIME: 2 HOURS**

Attempt ***question one (compulsory)*** and ***any other two*** questions.

QUESTION ONE (20 MARKS)

a) Evaluate $\lim_{x \to 1} \frac{x^3 - 1}{x - 1}$ (3 marks)

b) Consider the function $f(x) = \begin{cases} \frac{x^2 - 1}{x - 2} & x \neq 2 \\ 0 & x = -2 \end{cases}$

i. Sketch the graph of the function $f(x)$ (2 marks)

ii. Determine the points of discontinuity(if any) of the function $f(x)$ (3 marks)

c) Prove that $\frac{d}{dx}(\sec x) = \sec x \tan x$ (3 marks)

d) A projectile is fired straight upwards with a velocity of $400 m/s$; its distance above the ground t sec. after being fired is given by $S(t) = -16t^2 + 400t$. $S(t)$ is the distance of the particle from the ground after it has been fired. Calculate the maximum height achieved by the particle (3 marks)

e) Evaluate $\int \sin(3x-1)dx$ (3 marks)

f) Determine Taylors series expansion generated by the function $f(x)=\ln x$ upto the fifth term about $x=2$ (4 marks)

g) Calculate the volume obtained by rotating the area under the curve $y=1+x$ between $x=1$ and $x=2$ about the axis of x (4 marks)

h) Show that improper integral $\int_1^{\infty}\frac{1}{x}dx$ is divergent (5 marks)

QUESTION TWO (20 MARKS)

a) Consider the function $f(x)=x^3+x^2-5x-5$

b) Use the second derivative test to find the local maximum and minimum of the function $f(x)$ (7 marks)

c) Discuss the concavity of $f(x)$ (5 marks)

d) Determine the points of inflection of $f(x)$ (4 marks)

e) Sketch the graph of the function $f(x)$ (4 marks)

QUESTION THREE (20 MARKS)

a) A vessel containing water is in the form of an inverted hollow cone with semi-vertical angle of 30^0. There is a small hole at the vertex of the cone and the water is running out at a rate of $3cm^3$ per second $(3cm^3/s)$. Calculate the rate at which the surface area in contact with water is changing when there is $81\pi\ cm^3$ of water remaining in the cone.

(6 marks)

b) Evaluate $\int \frac{3x+7}{(x-1)(x^2+1)}dx$ (6 marks)

c) Prove that $\int \frac{dx}{x^2+a^2}=\frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right)$ (5 marks)

d) Calculate $\int e^{\sin x}\cos x dx$ (3 marks)

QUESTION FOUR (20 MARKS)

a) Determine $\frac{dy}{dx}$ for;

i. $y = \sinh x$ (2 marks)

ii. $y = \tanh x$ (2 marks)

iii. $y = \frac{(x^2+1)^5}{\sqrt{1+x}}$ (3 marks)

b) Calculate $\frac{d^2y}{dx^2}$ if;

i. $x = t - t^3 \quad ; \quad y = t - t^2$ (3 marks)

ii. $x = \cos t \quad ; \quad y = (1 - \sin^2 t)^{\frac{1}{2}}$ (3 marks)

c) Calculate the gradient of the curve $x^3 + 2x^2y^2 - 2y + xy = 2$ at the point $(-4, 1)$

(7 marks)

QUESTION FIVE (20 MARKS)

a) Evaluate

i. $\lim_{x \to 0^-} \frac{|x|}{x}$ (3 marks)

ii. $\lim_{x \to 2^+} \frac{(x^2+2)|x|}{x^2}$ (3 marks)

b) Consider the function $f(x) = \begin{cases} 7x - 2 & x \geq 2 \\ 3x + 5 & x < 2 \end{cases}$

i. Evaluate $\lim_{x \to 2^-} f(x)$ (3 marks)

ii. Sketch the graph of the function $f(x)$ (3 marks)

c) Calculate the slope of the tangent line to the curve $x^3 + 2xy - 3y^2 = 9$ at the point $(3, 2)$

(4 marks)

d) If $xy + y^2 = 1$ find;

i. $\frac{dy}{dx}$ (2 marks)

ii. $\frac{d^2y}{dx^2}$ (2 marks)