MACHAKOS UNIVERSITY

University Examinations 2022/2023 Academic Year

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

THIRD YEAR SECOND SEMESTER EXAMINATION FOR

BACHELOR OF ECONOMICS AND STATISTICS

SMA 361: THEORY OF ESTIMATION

DATE: 14/4/2023 **TIME: 2:00 – 4:00 PM**

INSTRUCTIONS

ANSWER QUESTION ONE AND OTHER TWO QUESTIOS

QUESTION ONE (COMPULSORY) (30 MARKS)

(a) $Let\ x_1, x_2, .. x_n$be identically independent random variable with a finite mean μ, show that the sample mean $\bar{x}$ is always unbiased estimator of μ. (4 marks)

(b) Let $x_1, x_2, .. x_n$ be a random sample of size n from a distribution with mean from a population with mean μ and variance δ^2 , show that $\bar{x}$ is a consistent estimator of μ. (6 marks)

(c) Let $x_1, x_2, .. x_n$ denote a random sample from a pdf is $f(x, \lambda) = \lambda x^{\lambda - 1}$, 0$< x < 1, \lambda > 0$ Show that $t_1 = \prod_{i-1}^{n} x_i$ is sufficient for λ (5 marks)

(d) Let $x_1, x_2, .. x_n$ be a random sample of size n from a normally distributed population with mean μ (unknown) and known variance δ^2.Determine the maximum likelihood estimator (MLE) of μ . (5 marks)

(e) $Let\ x_1, x_2, .. x_n$ be identically independent random variable with a finite variance δ^2 .Show that the sample variance $s^2 = \frac{1}{n-1}(\sum_1^n (x - \bar{x})^2$=$\frac{1}{n-1}\{\sum_1^n x^2 - n\bar{x}^2\}$ (5 marks)

(f) Let $x_1, x_2, ..x_n$ denote a random sample from a pdf

$$f(x) = \begin{cases} (\alpha+1)x^{\alpha}, & 0 < x < 1, \ \alpha > 0 \\ 0 & elsewhere \end{cases}$$

Obtain the moment estimator of α. (5 marks)

QUESTION TWO (20 MARKS)

(a) If $x_1, x_2, ..x_n$ is a random sample of size n from a population whose pdf is given by

$$f(x, \lambda) = \frac{e^{-\lambda}\lambda^x}{x!}, x = 0, 1, 2, ...$$

Determine the uniformly minimum variance unbiased estimator (UMVUE) of $e^{-\lambda}$

(10 marks)

(b) A random sample of size n is drawn from a normal population with $(0, \delta^2)$. Determine the minimum variance unbiased estimator (MVUE) of δ^2 and its variance. (10 marks)

QUESTION THREE (20 MARKS)

Three objects M, N and P with weights$t_{1,} t_2$ $and,$$t_3$ respectively are weighed on a spring balance M and N together weigh 75g, M and P weigh 45g and N and P weigh 100g. Assuming the weights are independent with constant variance δ^2, determine the least square estimates of $t_1, t_2, and\ t_3$

QUESTION FOUR (20 MARKS)

(a) Let $x_1, x_2, ..x_n$ be a random variable of size n from a population which is normally distributed with mean μ and δ^2 both unknown. Determine the MLE of μ and δ^2 (8 marks)

(b) A random sample $(X_1, X_2, X_3, X_4, X_5)$ of size 5 is drawn from a normal population with unknown mean μ. Consider the following estimators to estimate μ

$$t_1 = \frac{X_1 + X_2 + X_3 + X_4 + X_5}{5}$$

$$t_2 = \frac{X_1 + X_2}{2} + X_3$$

$$t_3 = \frac{2X_1 + X_2 + \lambda X_3}{3}$$

Where λ is such that t_3 is an unbiased estimator of μ

(i) Determine λ (4 marks)

(ii) Show that $t_1 and\ t_2$ are unbiased (4 marks)

(i) State giving reasons the estimators which is best among $t_1, t_2, and\ t_3$ (4 marks)

QUESTION FIVE (20 MARKS)

(a) A population has a known mean μ and a standard deviation δ of 2.45. If a sample of 900 has a mean of 3.15cm and the population is normal obtain a 98% confidence intervals for the true mean. (10 marks)

(b) Let X be random variable that is binomially distributed with a pdf defined as

$$f(x;\theta) = \binom{n}{x}\theta^x(1-\theta)^{n-x} \quad for\ x = 0,1 \dots n, \theta \in (0,1).$$

Determine the Crammer-Rao lower bound (10 marks)