MACHAKOS UNIVERSITY

University Examinations 2021/2022 Academic Year

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

SECOND YEAR SECOND SEMESTER EXAMINATION FOR

BACHELOR OF SCIENCE (APPLIED PHYSICS AND TECHNOLOGY)

SMA 263: COMPLEX ANALYSIS

DATE: 9/12/2021 **TIME: 8.30-10.30 AM**

INSTRUCTION:
Answer Question One and Any Other Two Questions

QUESTION ONE (30 MARKS) (COMPULSORY)

a) Given two complex numbers $z_1 = x_1 + iy_1$, $z_2 = x_2 + iy_2$ and $z = \frac{z_1}{z_2}$ determine $Re\, z$ and $Im\, z$. (4 marks)

b) Given the complex number $z = -\sqrt{3} - i$.

i. Represent z in the z-plane (2 marks)

ii. Express the number in polar form. (4 marks)

iii. Show that $z^3 = -8i$ (3 marks)

c) Determine the real and imaginary parts $u(x, y)$ and $v(x, y)$ of the complex function $f(z) = \frac{\overline{3z}}{z+1}$ (3 marks)

d) Construct the image S' of the square S with vertices at $1 + i, 2 + i, 2 + 2i$ and $1 + 2i$ under the linear mapping $T(z) = z + 2 - i$. (6 marks)

e) Evaluate $\oint_C \bar{z} dz$ where C is given by $x = 3t, y = t^2$ for $-1 \leq t \leq 4$. (5 marks)

f) Determine a linear fractional transformation that maps $z_1 = 1, z_2 = 0, z_3 = -1$ onto the points $w_1 = 2i, w_2 = \infty, w_3 = -1$. (3 marks)

QUESTION TWO (20 MARKS)

a) Evaluate the following;

i. $\lim_{z\to\infty} \frac{z^2+3z+2}{4z^2+2z-1}$ (3 marks)

ii. $\lim_{z\to i} \frac{(3+i)z^4-z^2+2z}{z+1}$ (3 marks)

b) Show that the function $f(z) = \frac{x}{x^2+y^2} - i\frac{y}{x^2+y^2}$ is analytic in any domain D that does not contain the point $z = 0$. (5 marks)

c) Show that $\oint_C \frac{5z+7}{z^2+2z-3} dz = 6\pi i$ where C is circle $|z-2| = 2$. (9 marks)

QUESTION THREE (20 MARKS)

a) Consider the complex function $f(z) = u(x,y) + iv(x,y)$ whereby

$u(x,y) = x^3 - 3xy^2 - 5y$;

i. Verify that $u(x,y)$ is harmonic in the entire complex plane. (3 marks)

ii. Find the harmonic conjugate function of $u(x,y)$. (6 marks)

b) Evaluate $\oint_C (x^2 + iy^2) dz$ where C is the contour starting at $z = 0$ passing through $z = 1 + i$ and ending at $z = 1 + 2i$. (11 marks)

QUESTION FOUR (20 MARKS)

a) Show that $(1+i)^8 = 16e^{2\pi i}$ (4 marks)

b) If $f(z) = z^2$ prove that $\lim_{z\to z_0} f(z) = z_0^2$, using the definition of limit. (6 marks)

c) Determine $\frac{1}{2\pi i}\oint_C \frac{e^{zt}}{z^2(z^2+2z+2)} dz$ around the circle $C: |z| = 3$ using the residue theorem.

(10 marks)

QUESTION FIVE (20 MARKS)

a) Determine a bilinear transformation that maps $z_1 = -1, z_2 = 0, z_3 = 1$ onto the points $w_1 = -i, w_2 = 1, w_3 = i$. (5 marks)

b) Express the following complex numbers in polar form $z_1 = 2 + i2\sqrt{3}, z_2 = -5 + i5$

(6 marks)

c) Consider a rectangular region R in the z-plane that is bounded by $x = 0, y = 0, x = 2, y = 1$. Determine the region R' in the w-plane onto which R is mapped by the transformation $w = \sqrt{2}\exp\left(\frac{\pi i}{4}\right)z$. (9 marks)