MACHAKOS UNIVERSITY

University Examinations for 2020/2021

SCHOOL OF ENGINEERING AND TECHNOLOGY

DEPARTMENT OF ELECTRICAL AND ELECTRONIC ENGINEERING

FOURTH YEAR FIRST SEMESTER EXAMINATION FOR

BACHELOR (SCIENCE IN TELECOMMUNICATION AND INFORMATION ENGINEERING

SPH 407: WAVEGUIDES

DATE: 17/8/2021 **TIME: 8:30 – 10:30 AM**

INSTRUCTIONS:

Answer Question One and Any Other Two Questions

QUESTION ONE-COMPULSORY (30 MARKS)

a) Explain why TEM mode of propagation is not supported in waveguides. (2 marks)

b) Copper has a conductivity of $5.8\text{x}10^7$ mhos/m and is considered an ideal material for electromagnetic shielding. A shield is made of copper with a thickness of 1 mm. If a uniform plane wave is normally incident upon the copper shield, determine, at 1 MHz and 500 MHz:

i) The attenuation constant
ii) The intrinsic impedance
iii) The absorption loss in the copper shield (9 marks)

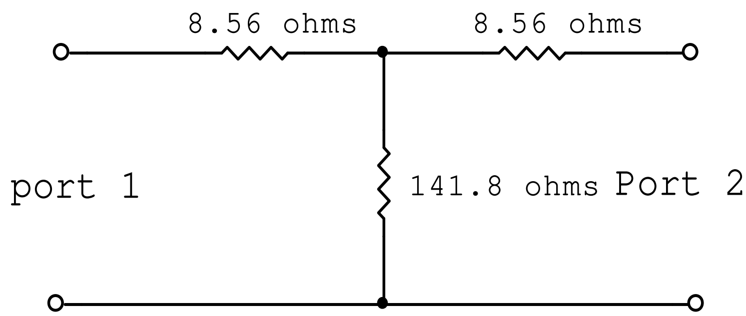
c) A uniform plane wave propagating in a medium has,

$$\boldsymbol{E} = \boldsymbol{6}e^{-\alpha z}\sin(10^8 t - \beta z)\,\boldsymbol{a}_y \quad V/m$$

If the medium is characterized by $\varepsilon_r = 1$, $\mu_r = 20$ and $\sigma = 3$ mhos/m, find the loss tangent, α, and β. (6 marks)

d) A standard air-filled rectangular waveguide with dimensions a = 8.636 cm, b = 4.318 cm is fed by a 4-GHz carrier from a coaxial cable. Determine if a TE_{10} mode will be propagated. If so, calculate the phase velocity and the group velocity. (5 marks)

e) Find the ***S*** parameters of the 3 dB attenuator circuit shown in the figure below



(4 marks)

f) A certain two-port network is measured and the following scattering matrix is obtained;

$$[S] = \begin{bmatrix} 0.1 < 0 & 0.8 < 90^0 \\ 0.8 < 90^0 & 0.2 < 0 \end{bmatrix}$$

From this data, determine whether the network is reciprocal or lossless. (4 marks)

QUESTION TWO (20 MARKS)

a) Derive and show that the scattering matrix, ***S*****,** of a directional coupler is given by

$$[S] = \begin{bmatrix} 0 & p & 0 & jq \\ p & 0 & jq & 0 \\ 0 & jq & 0 & p \\ jq & 0 & p & 0 \end{bmatrix}$$

Where $p^2 + q^2 = 1$ (10 marks)

b)

In an air-filled rectangular waveguide, the cutoff frequency of a TE_{10} mode is 5 GHz, whereas that of TE_{01} mode is 12 GHz. Calculate

i) The dimensions of the guide

ii) The cutoff frequencies of the next three higher TE modes

iii) The cutoff frequency for TE_{11} mode if the guide is filled with a lossless material having $\varepsilon_r = 2.25$ and $\mu_r = 1$. **(10 marks)**

QUESTION THREE (20 MARKS)

a) Dry ground has a conductivity of $5\text{x}10^{-4}$ mhos/m and a relative dielectric constant of 10 at a frequency of 500 MHz. Compute:
   i) The propagation constant
   ii) The intrinsic impedance
   iii) The phase velocity.

   Repeat for a frequency of 10 KHz. What does it imply about the ground as a conductor at very low frequencies? (10 marks)

b) Find the cut off frequencies of the first two propagating modes of a circular waveguide with $a = 0.5\ cm$ and $\varepsilon_r = 2.25$. If the guide is silver plated and the dielectric loss tangent is 0.001, calculate the attenuation in dB for a 50 cm length of guide operating at 13.0 GHz. (10 marks)

QUESTION FOUR (20 MARKS)

a) Given the four time varying Maxwell equations,

$$\overline{\nabla} \times \overline{E} = -\frac{\partial \overline{B}}{\partial t}$$

$$\overline{\nabla} \times \overline{H} = \overline{J} + \frac{\partial \overline{D}}{\partial t}$$

$$\overline{\nabla} . \overline{D} = \rho$$

$$\overline{\nabla} . \overline{B} = 0$$

Show that the propagation constant of plane electromagnetic wave is given by

$$\gamma = \sqrt{j\omega\mu(\sigma + j\omega\varepsilon)} = \alpha + j\beta$$

(10 marks)

b)

A manufacturer produces a ferrite material with $\mu = 750\mu_o$, $\varepsilon = 5\varepsilon_o$, and $\sigma = 10^{-6}$ S/m at 10 MHz.

i) Would you classify the material as lossless, lossy, or conducting?

ii) Calculate β and λ.

iii) Find the intrinsic impedance. **(10 marks)**

QUESTION FIVE (20 MARKS)

a) Derive and show that the scattering matrix, $\boldsymbol{S}$**,** of an E-plane TEE junction is given by:

$$[S] = \begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{12} & S_{11} & -S_{13} \\ S_{13} & -S_{13} & S_{33} \end{bmatrix}$$

(10 marks)

b)

A TE_{11} mode is propagating through a circular waveguide. The radius of the guide is 5 cm, and the guide contains an air dielectric (refer to Fig. 1).

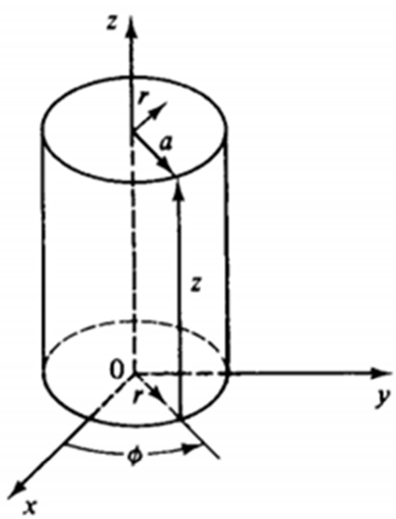


Fig 1

i). Determine the cutoff frequency.

ii). Determine the wavelength λ_g in the guide for an operating frequency of 3 GHz.

iii) Determine the wave impedance Z_g in the guide. **(10 marks)**

USEFUL INFORMATION

Constants

$c = 3\times 10^8$ m/s

$\epsilon_o = 8.85\times 10^{-12}$ F/m

$\mu_o = 4\pi\times 10^{-7}$ H/m

$\epsilon = \epsilon_o \epsilon_r$

$\mu = \mu_o \mu_r$

Maxwell's Equations

Differential form:

$$\nabla\times\vec{\varepsilon} = -\frac{\partial \vec{B}}{\partial t}$$

$$\nabla\times\vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}$$

$$\nabla\cdot\vec{D} = \rho$$

$$\nabla\cdot\vec{B} = 0$$

Integral Form:

$$\oint_c \vec{\varepsilon}.d\vec{l} = -\frac{\partial}{\partial t}\int_s \vec{B}.d\vec{s} - \int_s \vec{M}.d\vec{s}$$

$$\oint_c \vec{H}.d\vec{l} = \frac{\partial}{\partial t}\int_s \vec{D}.ds + \int_s \vec{J}.d\vec{s}$$

$$\oint_S \vec{B}.d\vec{s} = 0$$

$$\oint_S \vec{D}.d\vec{s} = \int_V \rho\, dv = Q$$

Lorentz's Force Equation

$\vec{F} = q(\vec{\varepsilon} + \vec{U}\times\vec{B})$ Newtons

Continuity Equation

$$\nabla\cdot\vec{J} = -\frac{\partial \rho}{\partial t}$$

General **Boundary conditions** between two media

$\vec{a}_n \times (\vec{\varepsilon}_2 - \vec{\varepsilon}_1) = 0$

$\vec{a}_n \times (\vec{H}_2 - \vec{H}_1) = \vec{J}_s$

$\vec{a}_n \cdot (\vec{D}_2 - \vec{D}_1) = \rho_s$

$\vec{a}_n \cdot (\vec{B}_2 - \vec{B}_1) = 0$

Plane Wave Propagation

Poynting's Vector

$\vec{S} = \vec{\varepsilon}\times\vec{H}$

The Wave equation

$$\nabla^2\vec{\varepsilon} = \mu\sigma\frac{\partial\vec{\varepsilon}}{\partial t} + \mu\in\frac{\partial^2\vec{\varepsilon}}{\partial t^2}$$

$$\nabla^2\vec{H} = \mu\sigma\frac{\partial\vec{H}}{\partial t} + \mu\in\frac{\partial^2\vec{H}}{\partial t^2}$$

Propagation constant

$$\gamma = \sqrt{j\omega\mu(\sigma + j\omega\in)}$$

Intrinsic impedance

$$\hat{\eta} = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\in}}$$

$$v_p = \frac{\omega}{\beta} \qquad \lambda_g = \frac{\lambda_o}{\sqrt{\epsilon_r}}$$

$$\beta = \frac{2\pi}{\lambda_g} \qquad v_g = \frac{d\omega}{d\beta}$$

Phasor Poynting Vector $\hat{S} = \hat{E} \times \hat{H}^*$

Average Power Flow $S_{av} = \frac{1}{2}\mathrm{Re}\left\{\hat{E} \times \hat{H}^*\right\}$

Skin Depth $\delta = \dfrac{1}{\sqrt{\pi f \mu \sigma}}$

Lossless medium	Good dielectric	Good Conductor
$\alpha = 0$	$\alpha = \frac{\sigma}{2}\sqrt{\frac{\mu}{\epsilon}}$	$\alpha \cong \sqrt{\frac{\omega\mu\sigma}{2}}$
$\beta = \omega\sqrt{\mu\epsilon}$	$\beta \cong \omega\sqrt{\mu\epsilon}$	$\beta \cong \sqrt{\frac{\omega\mu\sigma}{2}}$
$v_p = 1/\sqrt{\mu\epsilon}$	$v_p \cong 1/\sqrt{\mu\epsilon}$	$v_p \cong \sqrt{\frac{2\omega}{\mu\sigma}}$
$\eta = \sqrt{\frac{\mu}{\epsilon}}$	$\eta \cong \sqrt{\frac{\mu}{\epsilon}}$	$\hat{\eta} = \sqrt{\frac{\omega\mu}{\sigma}}\angle 45^\circ$

$$Z_o = \sqrt{\frac{R + j\omega L}{G + j\omega C}}$$

$$V(z) = V_o^+\left[e^{-j\beta z} + \Gamma e^{j\beta z}\right]$$

$$I(z) = \frac{V_o^+}{Z_o}\left[e^{-j\beta z} - \Gamma e^{j\beta z}\right]$$

$$\Gamma = \frac{V_o^-}{V_0^+} = \frac{Z_L - Z_o}{Z_L + Z_o}$$

RL = $-20\log|\Gamma|$ dB

$$VSWR = \frac{1+|\Gamma|}{1-|\Gamma|}$$

$$Z_{in} = Z_o \frac{Z_L + jZ_o \tan\beta l}{Z_o + jZ_L \tan\beta l}$$

EMC

$$E_{out} = \tau_{eff} E_{in}$$

$$\tau_{eff} \approx \frac{1}{1 + {z_0}/{(2R_s)}} \qquad where\ \tau_{eff} = \frac{\tau_{12}\tau_{23}e^{-\gamma L}}{1 - \rho_{21}\rho_{23}e^{-2\gamma L}}$$

$$where\ R_s = \frac{1}{\sigma L} = surface\ resistance$$

the sheet

For Electric fields through a shielding block

For very thin films L<<skin depth

TABLE 4-2-2 pth ZEROS OF $J_n(K_c a)$ FOR TM_{np} MODES

p	$n =$ 0	1	2	3	4	5
1	2.405	3.832	5.136	6.380	7.588	8.771
2	5.520	7.106	8.417	9.761	11.065	12.339
3	8.645	10.173	11.620	13.015	14.372	
4	11.792	13.324	14.796			

TABLE 4-2-1 pth ZEROS OF $J_n'(K_c a)$ FOR TE_{np} MODES

p	$n =$ 0	1	2	3	4	5
1	3.832	1.841	3.054	4.201	5.317	6.416
2	7.016	5.331	6.706	8.015	9.282	10.520
3	10.173	8.536	9.969	11.346	12.682	13.987
4	13.324	11.706	13.170			

$$(\vec{\mathbf{A}}\times\vec{\mathbf{B}})\cdot\vec{\mathbf{C}}=(\vec{\mathbf{B}}\times\vec{\mathbf{C}})\cdot\vec{\mathbf{A}}=(\vec{\mathbf{C}}\times\vec{\mathbf{A}})\cdot\vec{\mathbf{B}}$$

$$\vec{\mathbf{A}}\times(\vec{\mathbf{B}}\times\vec{\mathbf{C}})=(\vec{\mathbf{A}}\cdot\vec{\mathbf{C}})\vec{\mathbf{B}}-(\vec{\mathbf{A}}\cdot\vec{\mathbf{B}})\vec{\mathbf{C}}$$

$$\nabla\cdot\left(\vec{\mathbf{A}}+\vec{\mathbf{B}}\right)=\nabla\cdot\vec{\mathbf{A}}+\nabla\cdot\vec{\mathbf{B}}$$

$$\nabla(\mathbf{V}+\mathbf{W})=\nabla\mathbf{V}+\nabla\mathbf{W}$$

$$\nabla\times(\vec{\mathbf{A}}+\vec{\mathbf{B}})=\nabla\times\vec{\mathbf{A}}+\nabla\times\vec{\mathbf{B}}$$

$$\nabla\cdot(\mathbf{V}\vec{\mathbf{A}})=\vec{\mathbf{A}}\cdot\nabla\mathbf{V}+\mathbf{V}\nabla\cdot\vec{\mathbf{A}}$$

$$\nabla\times\left(\mathbf{V}\vec{\mathbf{A}}\right)=\nabla\mathbf{V}\times\vec{\mathbf{A}}+\mathbf{V}\nabla\times\vec{\mathbf{A}}$$

$$\nabla\left(\vec{\mathbf{A}}\cdot\vec{\mathbf{B}}\right)=(\vec{\mathbf{A}}\cdot\nabla)\vec{\mathbf{B}}+(\vec{\mathbf{B}}\cdot\nabla)\vec{\mathbf{A}}+\vec{\mathbf{A}}\times(\nabla\times\vec{\mathbf{B}})+\vec{\mathbf{B}}\times(\nabla\times\vec{\mathbf{A}})$$

$$\nabla\times(\vec{\mathbf{A}}\times\vec{\mathbf{B}})=\vec{\mathbf{A}}\nabla\cdot\vec{\mathbf{B}}-\vec{\mathbf{B}}\nabla\cdot\vec{\mathbf{A}}+(\vec{\mathbf{B}}\cdot\nabla)\vec{\mathbf{A}}-(\vec{\mathbf{A}}\cdot\nabla)\vec{\mathbf{B}}$$

$$\nabla\cdot\nabla\mathbf{V}=\nabla^2\mathbf{V}$$

$$\nabla\cdot\nabla\times\vec{\mathbf{A}}=0$$

$$\nabla\times\nabla\mathbf{V}=0$$

$$\nabla\times\nabla\times\vec{\mathbf{A}}=\nabla(\nabla\cdot\vec{\mathbf{A}})-\nabla^2\vec{\mathbf{A}}$$

CONDUCTIVITIES OF MATERIALS

Material	Conductivity, S/m	Material	Conductivity, S/m
Copper (annealed)	5.8×10^7	Steel	$0.5\text{–}1.0 \times 10^7$
Aluminum	3.54×10^7	Water (distilled)	2×10^{-4}
Silver	6.14×10^7	Sea water	3–5
Nickel	1.28×10^7	Quartz (fused)	$< 2 \times 10^{-17}$

DIELECTRIC CONSTANTS OF MATERIALS

Material	*Frequency, MHz*	ϵ'/ϵ_0	*Loss tangent* ϵ''/ϵ'
Polystyrene	3,000	2.54	0.00025–0.0016
Polystyrene (foam)	3,000	1.05	0.00003
Lucite	10,000	2.56	0.005
Teflon	10,000	2.08	0.00037
Fused quartz	10,000	3.78	0.0001
Ruby mica	3,000	5.4	0.0003
Titanium dioxide	10,000	90	0.002
Mahogany wood	10,000	1.7	0.021

SKIN DEPTH IN COPPER

Frequency, Hz	10	60	10^2	10^3	10^4	10^8
Skin depth δ_s, cm	2.08	0.85	0.66	0.208	6.6×10^{-2}	6.6×10^{-4}

$\delta_s = \sqrt{2/\omega\mu\sigma} = 6.6 f^{-1/2}$ cm for copper ($\sigma = 5.8 \times 10^7$ S/m).