MACHAKOS UNIVERSITY

University Examinations 2018/2019

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS, STATISTICS AND ACTUARIAL SCIENCE

FOURTH YEAR SUPPLEMENTARY EXAMINATION FOR

BACHELOR OF SCIENCE (MATHEMATICS AND COMPUTER SCIENCE)

BACHELOR OF SCIENCE (MATHEMATICS)

BACHELOR OF EDUCATION

BACHELOR OF ARTS

SMA 431: DIFFERENTIAL GEOMETRY

DATE: 22/7/2019 **TIME: 8:30 – 10:30 AM**

INSTRUCTIONS:

Answer question one and any other two

QUESTION ONE (30 MARKS)

a) Find a unit vector normal to the plane S containing $P(0,1,1)$, $Q(1,0,-1)$, $R(1,-1,0)$ (3 marks)

b) Show that $\left||a|-|b|\right| \leq |a \pm b|$ for all a and b (4 marks)

c) Show that $x = te_1 + (t^2+1)e_2 + (t-1)e_3$ is a regular parametric representation for all t and find projections onto the x_1x_2 and x_1x_3 planes (4 marks)

d) Find the equation of the line tangent to the curve generated by $x = te_1 + t^2e_2 + t^3e_3$ at $t = 1$ (4 marks)

e) Compute the length of the arc $x = 3(\cosh 2t)e_1 + 3(\sinh 2t)e_2 + 6te_3$, $0 \leq t \leq \pi$ (4 marks)

f) Let $f(t) = (1+t^3)e_1 + (2t-t^2)e_2 + te_3$, $g(t) = (1+t^2)e_1 + t^3e_2$. Find:

a) $f(a) \bullet g(b)$ (2 marks)

b) $f(t) \times g(t)$ (2 marks)

g) Find the curvature vector k and curvature $|\kappa|$ on the curve $x = te_1 + \frac{1}{2}t^2e_2 + \frac{1}{3}t^3e_3$ at the point $t = 1$ (7 marks)

QUESTION TWO (20 MARKS)

a) Using the definition of the limits, show that

$\lim_{t \to 1}\left(t^2e_1 + (t+1)e_2\right) = e_1 + 2e_2$ (4 marks)

b) Let $u = (\sin t)e_1 + 2t^2 + te_3$, $(t > 0)$, and $t = \log\theta$, find $\frac{du}{d\theta}$ as a:

   a) Function of θ (3 marks)

   b) Function of t (3 marks)

c) Let $a = e_1 - 2e_2 + 3e_3$ and $b = 2e_1 - 3e_2 + e_3$.

   a) Show that b is in $S_3(a)$ (2 marks)

   b) Find a $\delta > 0$ such that $S_\delta(b)$ is contained in $S_3(a)$ (4 marks)

   c) Find ε_1 and ε_2 such that $S_{\varepsilon_1}(a)$ and $S_{\varepsilon_2}(b)$ are disjoint (4 marks)

QUESTION THREE (20 MARKS)

a) Evaluate $\lim_{t \to 2}\left[(3t^2+1)e_1 - t^3e_2 + e_3\right]$ (3 marks)

b) Show that the curve generated by;

$x = (-1 + \sin 2t \cos 3t)e_1 + (2 + \sin 2t \sin 3t)e_2 + (-3 + \cos 2t)e_3$

Lies on the sphere of radius 1 about $a = -e_1 + 2e_2 - 3e_3$ (4 marks)

c) Find the equation of the tangent line and normal to the curve $x = (1+t)e_1 - t^2e_2 + (1+t^3)e_3$ at $t = 1$ (5 marks)

d) If $u = (3t^2+1)e_1 + (\sin t)e_2$ and $v = (\cos t)e_1 + e^te_3$. Find:

   a) $\frac{d}{dt}(u \bullet v)$ (3 marks)

   b) $\frac{d}{dt}(u \times v)$ (3 marks)

QUESTION FOUR (20 MARKS)

a) Let $f(t) = (\sin t)e_1 + te_3$ and $g(t) = (t^2+1)e_1 + e^te_2$. Find:

   a) $\lim_{t \to 0}\left(f(t) \bullet g(t)\right)$ (2 marks)

   b) $\lim_{t \to 0}\left(f(t) \times g(t)\right)$ (2 marks)

b) Show that the representation $x_1 = (1+\cos\theta)$, $x_2 = \sin\theta$, $x_3 = 2\sin(\theta/2)$, $-2\pi \le \theta \le 2\pi$ is a regular and lies on then sphere of radius 2 about the origin and the cylinder $(x_1 - 1)^2 + x_2^2 = 1$ (4 marks)

c) Prove the *Cauchy-Schwartz* inequality, $|a \bullet b| \le |a||b|$, with equality holding if and only if a and b are linearly dependent (4 marks)

d) Find the arc length as a function of θ along the epicycloid: $x_1 = (r_0 + r)\cos\theta - r\cos\left(\frac{r_0 + r}{r}\theta\right)$,

$x_2 = (r_0 + r)\sin\theta - r\sin\left(\frac{r_0 + r}{r}\theta\right)$ (8 marks)

QUESTION FIVE (20 MARKS)

a) Let $u = a(\cos t)e_1 + a(\sin t)e_2 + bte_3$, $a, b \ne 0$. Find:

a) $\frac{d^2u}{dt^2}$ (2 marks)

b) $\left|\frac{d^2u}{dt^2}\right|$ (2 marks)

b) Let the curve be $x = (3t - t^3)e_1 + 3t^2 e_2 + (3t + t^3)e_3$

i) Find a continuous unit principal normal and unit binormal along the curve (10 marks)

ii) Find the curvature and torsion (6 marks)