



MACHAKOS UNIVERSITY

University Examinations 2021/2022 Academic Year

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

THIRD YEAR SECOND SEMESTER EXAMINATION FOR

BACHELOR OF SCIENCE (MATHEMATICS AND COMPUTER SCIENCE)

BACHELOR OF SCIENCE (MATHEMATICS)

BACHELOR OF EDUCATION(SCIENCE)

BACHELOR OF EDUCATION (ARTS)

BACHELOR OF ARTS

SMA 332: METHODS OF APPLIED MATHEMATICS

DATE: 8/12/2021

TIME: 8.30-10.30 AM

INSTRUCTION:

Answer Question One and Any Other Two Questions

QUESTION ONE (30 MARKS) (COMPULSORY)

- a) Decompose the equation $u_{tt} = c^2 u_{xx}$, $0 < x < L$, $t > 0$ into ordinary differential equations (ODEs) in variables X and T . (6 marks)
- b) State the Fourier series expansion of a function $f(x)$ over the interval $-\pi < x < \pi$. (2 marks)
- c) Show that the Fourier coefficient a_n of the Fourier series, you have defined in b) above is given by; $a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos nx \, dx$, $n = 1, 2, 3, \dots$ (5 marks)
- d) Determine the Laplace transform of $f(x) = \cos ax$. (5 marks)
- e) Use the Convolution theorem to determine the inverse Laplace transform of the quotient
$$\frac{6}{(s^2 + 1)(s^2 + 9)}$$
 (6 marks)
- f) Show that the Laplace transform $\frac{\partial^2 u}{\partial t^2}$ is given by $s^2 U(x, s) - su(x, 0) - u_t(x, 0)$ (6 marks)

QUESTION TWO (20 MARKS)

- a) Classify the 2nd order partial differential equation (PDE) $y^2 u_{xx} + x^2 u_{yy} = 0$ and transform it to canonical form. (13 marks)
- b) Obtain the Fourier series for the function

$$\begin{aligned} f(x) &= 2x + 1, & -\pi < x < \pi \\ f(x) &= f(x + 2\pi) \end{aligned} \quad (7 \text{ marks})$$

QUESTION THREE (20 MARKS)

- a) Use partial fractions to determine the inverse Laplace transform of the quotient:

$$\frac{s^2}{(s+3)^3} \quad (8 \text{ marks})$$
- b) Solve the following 2nd differential equation using Laplace transform method;
 $y'' + 5y' + 6y = 2e^{-t}$ subject to $y(0) = 1, y'(0) = 0$ (12 marks)

QUESTION FOUR (20 MARKS)

- a) A heavy-duty wire of length a that is fixed at $x = 0$ and free to move at the other end $x = a$ is initially at rest and subjected to a constant force. Assuming that the wire's displacement $u(x, t)$ happens only in the x direction and $u_x(a, t) = D$ where D is the displacement per unit length.
- i) Show that the Laplace transform of the associated wave equation is given by the ODE

$$\frac{d^2 U(x, s)}{dx^2} = \frac{s^2}{c^2} U(x, s) \quad (5 \text{ marks})$$
- ii) Show that the solution to the ODE given in 4 a) i) above is

$$U(x, s) = \frac{cD \sinh(sx/c)}{s^2 \cosh(sa/c)} \quad (10 \text{ marks})$$
- b) Show that $\int_{-1}^1 \cos m\pi x \sin n\pi x dx = 0$ where m, n are integers. (5 marks)

QUESTION FIVE (20 MARKS)

- a) A thin bar of length π units is placed in boiling water (temperature 100°C). After reaching 100°C throughout, the bar is removed from the boiling water. With the lateral sides kept insulated, suddenly, at time $t = 0$, the ends are immersed in a medium with constant freezing temperature 0°C . Taking $c=1$, find the temperature $u(x, t)$ for $t > 0$ using the separation of variables method (10 marks)
- b) Obtain the general solution to the 2nd order PDE $y u_{tt} - k^2 y u_{yy} = 2k^2 u_r$ where k is a constant. (10 marks)