MACHAKOS UNIVERSITY

UNIVERSITY EXAMINATIONS 2018/2019

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS, STATISTICS AND ACTUARIAL SCIENCE

SECOND YEAR FIRST SEMESTER EXAMINATION FOR

BACHELOR OF SCIENCE (MATHEMATICS AND COMPUTER SCIENCE)

BACHELOR OF EDUCATION (ARTS)

BACHELOR OF EDUCATION (SCIENCE)

SMA 231: DYNAMICS I

DATE: 30/4/2019 **TIME: 2:00 – 4:00 PM**

INSTRUCTION: *Answer Question **ONE** which is compulsory*

QUESTION ONE (COMPULSORY) (30 MARKS)

a) If $A = A_1\boldsymbol{i} + A_2\boldsymbol{j} + A_3\boldsymbol{k}$ and $B = B_1\boldsymbol{i} + B_2\boldsymbol{j} + B_3\boldsymbol{k}$ prove that

$A.B = A_1B_1 + A_2B_2 + A_3B_3$ (3 marks)

b) If $\phi = x^2yz^3$ and $A = xz\boldsymbol{i} - y^2\boldsymbol{j} + 2x^2y\boldsymbol{k}$, find $\nabla.(\phi A)$ (3 marks**)**

c) Due to a force field, a particle of mass 5 units moves along a space curve whose position vector is given as a function of time t by

$\boldsymbol{r} = (2t^3 + t)\boldsymbol{i} + (3t^4 - t^2 + 8)\boldsymbol{j} - 12t^2\boldsymbol{k}$ find;

i) Velocity (2 marks)

ii) Momentum (2 marks)

iii) Acceleration (2 marks)

iv) Force (2 marks)

v) The instantaneous power applied to the particle by the force field (2 marks)

d) A particle of mass m moves in the $xy-plane$ so that its position vector is $\boldsymbol{r} = a\cos\omega t\,\boldsymbol{i} + b\sin\omega t\,\boldsymbol{j}$ where $a, b\ and\ \omega$ are positive constants and $a > b$

i) Show that the particle moves in an ellipse (4 marks)

ii) Show that the force acting on the particle is always directed towards the origin

(4 Marks)

e) A particle of constant mass m moves in space under the influence of a force field $\boldsymbol{F}$. Assuming that at times t_1 and t_2 the velocity is v_1 and v_2 respectively, prove that the work done is the change in kinetic energy

$i.e$

$$\int_{t_1}^{t_2} \boldsymbol{F}.d\boldsymbol{r} = \frac{1}{2}m{v_2}^2 - \frac{1}{2}m{v_1}^2$$

(2 marks)

f) Show that the force field, $\boldsymbol{F}$ defined by $\boldsymbol{F} = (y^2z^3 - 6xz^2)\boldsymbol{i} + 2xyz^3\boldsymbol{j} + (3xy^2z^2 - 6x^2z)\boldsymbol{k}$ is a conservative force field. Hence find the work done by the force field $\boldsymbol{F}$ in moving a particle from the point $A(-2,1,3)$ to $B(1,-2,-1)$ (4 marks)

QUESTION TWO (20 MARKS)

a) A particle of mass m moves along a straight line under the influence of a constant force of magnitude F . if its initial speed is v_0 find;

i) The speed (3 marks)

ii) The velocity (2 marks)

iii) The distance travelled after time, t (3 marks)

iv) Show that the speed of the particle at any position x is given by

$v = \sqrt{{v_0}^2 + (2F/m)x}$ (4 marks)

b) An object of mass m is thrown vertically upwards from the earth's surface with speed v_0 find;

i) The position at any time (2 marks)

ii) The time taken to reach the highest point (1 mark)

iii) The maximum height reached (2 marks)

iv) The speed of the particle in terms of its distance from the origin O

(3 marks)

QUESTION THREE (20 MARKS)

a) A projectile is launched with initial speed v_0 at an angle $\propto$ with the horizontal, find;

i) The position vector at any time **(**2 marks)

ii) The time to reach the highest point (2 marks)

iii) The maximum height reached (2 marks)

iv) The time of flight back to the earth (2 marks)

v) The range (2 marks)

vi) Show that the path of the projectile is a parabola (2 marks)

vii) Prove that the range of the projectile is a maximum when the launching angle $\propto = 45°$ (2 marks)

b) Considering a uniform force field, $\boldsymbol{F} = -F_0\boldsymbol{k}$;

i) Prove that the uniform force field is conservative (2 marks)

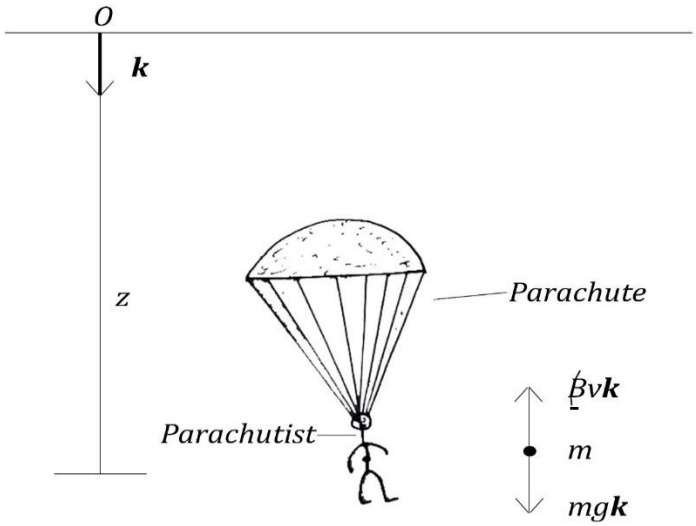
ii) Find the potential corresponding to this field (2 marks)

iii) Deduce the potential energy of a particle of mass m in a uniform gravitational force field (2 marks)

QUESTION FOUR (20 MARKS)

a) At time $t = 0$ a parachutist shown in figure 1 below, having weight of magnitude mg is located at $z = 0$ and is travelling vertically downward with speed v_0 .

Figure 1.



If the force or air resistance acting on the parachute is proportional to the instantaneous speed find;

i) The speed (4 marks)

ii) Distance travelled (3 marks)

iii) The acceleration at any time $t > 0$ (2 marks)

iv) Show that the parachutist approaches a limiting speed given by mg/β (2 marks)

b) Given the force field $\boldsymbol{F} = (2xy + z^3)\boldsymbol{i} + x^2\boldsymbol{j} + 3xz^2\boldsymbol{k}$;

i. Show that $\boldsymbol{F}$ is a conservative force field. (2 marks)

Hence or otherwise find;

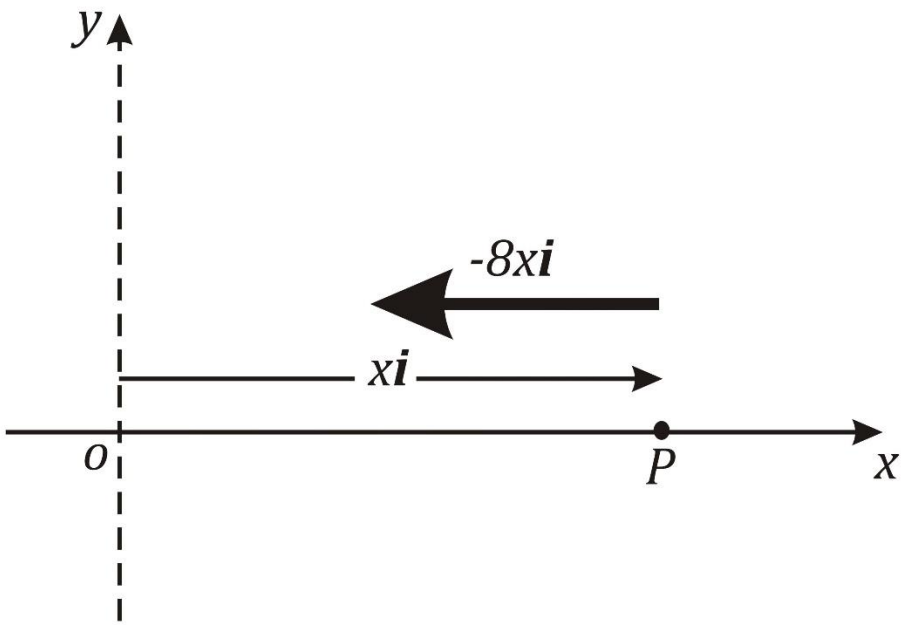
ii. The potential (3 marks)

iii. The Work done in moving an object in this field from $(1, -2, 1)$ to $(3,1,4)$ (4 marks)

QUESTION FIVE (20 MARKS)

a) A particle P of mass 2 moves along the $x - axis$ attracted toward origin O by a force whose magnitude is numerically equal to $8x$ as shown in figure 2 below.

Figure 2.



If it is initially at rest at $x = 20$ find;

i) The differential equation and initial conditions describing the motion (3 marks)

ii) The position of the particle at any time (3 marks)

iii) The speed and velocity of the particle at any time (1 mark)

iv) The amplitude, period and frequency of the vibration (3 marks)

b) An object of mass $20\ kg$ moves with simple harmonic motion on the $x - axis$. Initially $(t = 0)$ it is located at the distance 4 metres away from the origin $x = 0$ and has velocity $15\ m/s$ and acceleration $100\ m/s^2$ directed toward $x = 0$. Find;

i) The position at any time (4 marks)

ii) The amplitude, period and frequency of the oscillations (3 marks)

iii) The force on the object when $t = \pi/10$ second (3 marks)