MACHAKOS UNIVERSITY

ISO 9001:2008 Certified

School of Education for the Bachelor's Degree

Second year Supplementary examinations 2018

SMA 231: Dynamics I

Department of Mathematics and Statistics.

Time 2hours

Attempt question ***one (compulsory***) and any other ***two questions***.

Question 1(30mks)

a.) Define the following terms as used in theoretical Dynamics;

i.) A projectile (1mk)

ii.) Simple harmonic motion (1mk)

iii.) Momentum of a moving particle (1mk)

iv.) An Energy curve (1mk)

b.) A particle of mass 2 units moves in a force field depending on time (t) given by

$\vec{F} = 24t\hat{i} + (36t - 16)\hat{j} - 12t\hat{k}$. Determine;

i.) Torque (5mks)

ii.) Angular momentum about the origin at any time t, assuming that at $t = 0$, the particle is located at $\vec{r}_0 = 3\hat{i} - \hat{j} + 4\hat{k}$ and has velocity $\vec{v} = 6\hat{i} + 15\hat{j} - 8\hat{k}$ (5mks)

c.) A particle P of mass M units slides without rolling down a frictionless inclined plane AB of angle θ with the horizontal. If it starts from rest at the top A of the incline; calculate the particles acceleration (5mks)

d.) Show that the force field defined by $\vec{F} = \left(y^2 z^3 - 6xz^2\right)\hat{i} + 2xyz^3\,\hat{j} + \left(3xy^2 z^2 - 6x^2 z\right)\hat{k}$ is conservative. (5mks)

e.) A projectile having horizontal range R reaches a maximum height H. prove that it must have been launched with an initial speed equal to $\sqrt{\frac{g\left(R^2 + 16H^2\right)}{8H}}$ (6mks)

Question 2 (20mks)

a.) A particle of mass 4 units moves in such a way that it describes a space curve defined by $\vec{r} = \left(8t^4 - t^2\right)\hat{i} - 6t^2\,\hat{j} + \left(t^5 - 4\right)\hat{k}$. Calculate its;

i.) Velocity (2mks)

ii.) Momentum at $t = 1$ sec . (3mks)

iii.) Acceleration at $t = 1$ sec (3mks)

iv.) Force field at any time t (2mks)

b.) A particle of mass M units moves in the $xy-plane$ so that its position vector is $\vec{r} = a\cos\omega t\hat{i} + b\sin\omega t\hat{j}$ where $a, b\ and\ \omega$ are positive constants and $a > b$;

i.) Show that the particle moves in an ellipse (4mks)

ii.) Show that the force acting on the particle is always directed towards the origin (6mks)

Question 3(20mks)

a.) A 20kg mass (M1) moves east with a velocity of $0.5m/s$ along a frictionless surface towards another 20kg mass (M2) attached to a spring with a spring constant of $5.0N/m$.

i.) Determine the total kinetic energy of the system after the impact (2mks)

ii.) Describe the action after M1 collides elastically with M2 (3mks)

iii.) How much does M2 compress the spring before sloping (2mks)

b.) Suppose that a particle has constant mass and that at times t_1 *and* t_2 it is located at p_1 *and* p_2 respectively and moving with velocities $\vec{v}_1 = \frac{d\vec{r}_1}{dt}$ *and* $\vec{v}_2 = \frac{d\vec{r}_2}{dt}$ respectively; prove that the total work done in moving the particle a long a curve from p_1 *to* p_2 is given by;

$$W = \int_c \vec{f} \cdot d\vec{r} = \frac{1}{2} m\left(\vec{v}_2^{\,2} - \vec{v}_1^{\,2}\right) \quad \text{(6mks)}$$

c.) A particle of mass M moves along the $x-axis$ under the influence of a conservative force field having potential $\varphi(x)$. If the particle is located at the position x_1 *and* x_2 at the respective times t_2 *and* t_2. Prove that E is the total energy, the;

$$t_2 - t_1 = \sqrt{\frac{m}{2}} \int_{x_1}^{x_2} \frac{dx}{\sqrt{E - \varphi(x)}} \quad \text{(7mks)}$$

Question 4(20mks)

a.) A particle P is projected with an initial velocity of $20m/s$ at 75^0 to the horizontal from a point O at the bottom of a plane inclined at 45^0 to the horizontal. The particle hits the plane again at a point A where OA is a line of greatest slope. Calculate;

i.) The time of flight (6mks)

ii.) The range on the plane (3mks)

iii.) The greatest height above the plane reached by the particle (5mks)

b.) An object slides on a space /surface of ice along the horizontal straight line OA. At a certain point in its path the speed is v_0 and the object then comes to rest after travelling a distance x_0 . prove that the coefficient of friction is $\frac{v_0^2}{2gx_0}$ (6mks)

Question 5(20mks)

a.) An object of mass $40kg$ moves with simple harmonic motion on the $x-axis$, initially its located at the distance $4m$ away from the origin $(x=0)$ and have velocity $15m/s$ and acceleration of $100m/s^2$ directed towards $x=0$. Determine;

i.) The position at any time t (6mks)

ii.) Amplitude of the oscillation (2mks)

iii.) Frequency of the oscillation (2mks)

iv.) Period of the oscillation (2mks)

v.) Force on the object when $t=\frac{\pi}{20}\sec$ (5mks)

b.) Derive the equation for energy of a simple Harmonic Oscillation

$E=\frac{1}{2}mv^2+\frac{1}{2}kx^2$ (3mks)