MACHAKOS UNIVERSITY

University Examinations 2022/2023 Academic Year

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

THIRD YEAR SUPPLEMENTARY EXAMINATION FOR

BACHELOR OF ARTS

SMA 336: ORDINARY DIFFERENTIAL EQUATION II

DATE: 10/3/2023 **TIME: 2:00-4:00 P.M**

INSTRUCTION:

Answer Question One and Any Other Two Questions

QUESTION ONE (COMPULSORY) (30 MARKS)

a) Determine the singular points for the differential equation $x^2(x-2)^2\frac{d^2y}{dx^2}+2(x-2)\frac{dy}{dx}+(x+1)y=0$ (3 marks)

b) Prove the Bessel function $J_{-\frac{1}{2}}(x)=\sqrt{\frac{2}{\pi x}}\cos x$ (4 marks)

c) Consider the differential equation

$(z+z^3)\cos x\ dx-(z+z^3)dy+(1-z^2)\ (y-\sin x)dz=0$.

Solve the equation using Netani's method. (4 marks)

d) Consider the initial – value problem $\frac{dy}{dx}=x^2+y^2$, $y(1)=3$. Determine whether the differential equation has a unique solution (4 marks)

e) Solve the system of differential equations below using the elimination method

$$\frac{dy}{dx}=x+2y+1$$

$$\frac{dy}{dx}=3x+2y+t$$

(5 marks)

(5 marks)

f) Determine the integrability of the differential equation given by; $2xy(y-z)dx - z(x+2z)dy + y(x^2+2y)dz = 0$ (5marks)

QUESTION TWO (20MARKS)

a) Define the Legendre equation of order p (3 marks)

b) Show that the Legendre equation $(1-x^2)y'' - 2xy' + p(p+1)y = 0$ where p is a constant, has a regular singularity as $|x| \to \infty$ (8 marks)

c) Use the method of Frobenius to solve the differential equation

$$2x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + (x^2-3)y = 0$$ (9 marks)

QUESTION THREE (20 MARKS)

a) Define a Bessel equation of order p (3 marks)

b) Determine the general solution for the Bessel equation $x^2y'' + xy' + (x^2-5)y = 0$ (4 marks)

c) Evaluate the Bessel function integral $\int J_5(x)dx$ (4 marks)

d) Prove that $J_{\frac{1}{2}}(x) = \sqrt{\frac{2}{\pi x}}\sum_{k=0}^{\infty}\frac{(-1)^k x^{2k+1}}{(2x+1)}$ (9 marks)

QUESTION FOUR (20 MARKS)

a) Use the Netani's method to solve the differential equation

$(2x^2 + 2xy + 2xz^2 + 1)dx + dy + 2zdz = 0$ (9 marks)

b) Solve the system

$$\frac{dx}{dt} = 2x + y + 3e^{2t}$$
$$\frac{dy}{dt} = -4x + 2y + t\,e^{2t}$$ (11 marks)

QUESTION FIVE (20 MARKS)

a) Define the following terms

i. Boundary value problem (2 marks)

ii. General solution of an n^{th} order differential equation (2 marks)

iii. Particular solution of n^{th} order differential equation (2 marks)

b) Consider the differential equation

$$\frac{d^3z}{dt^3} - 3\frac{d^2z}{dt^2} + 2\frac{dz}{dt} - z = \sin t \;\; ; \;\; z(0) = 1, z'(0) = 0 \; , \; z''(0) = 3$$

Reduce the equation to a system of first order differential equation (5 marks)

c) Use the matrix method to solve the system

$$\begin{pmatrix} x \\ y \end{pmatrix}' = \begin{pmatrix} 8 & -1 \\ 4 & 12 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix}$$ (9 marks)