MACHAKOS UNIVERSITY

University Examinations 2019/2020 Academic Year

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

THIRD YEAR SPECIAL /SUPPLEMENTARY EXAMINATION FOR

BACHELOR OF SCIENCE IN STATISTICS AND PROGRAMMING

BACHELOR OF SCIENCE INMATHEMATICS

BACHELOR OF EDUCATION

SMA 465: MEASURE AND PROBABILITY

DATE: 20/1/2021 **TIME: 2.00-4.00 PM**

INSTRUCTIONS

Answer question ONE and any other TWO questions

QUESTION ONE

a) Let $\mathbf{\Omega}$ be a non-empty set, $\boldsymbol{\pi}$ be a $\boldsymbol{\delta}$-algebra on $\mathbf{\Omega}$ and $\boldsymbol{\mu}$ be a measure defined on $\boldsymbol{\pi}$. Define;

i. A measurable space

ii. A measure space

iii. A complete measure space

iv. $\boldsymbol{\mu}$- null set

v. $\boldsymbol{\mu}$ -almost everywhere (10 marks)

b) Let $\{ \mathbf{\Omega} , \boldsymbol{\pi}, \mathbf{P} \}$ be a probability measure space and $\mathbf{A}_j \in \boldsymbol{\pi}$ for j=1,2,3,…. Show that; $\mathbf{P}(\mathbf{A}_1) \leq \mathbf{P}(\mathbf{A}_2)$, if $\mathbf{A}_1 \subset \mathbf{A}_2$ (4 marks)

c) Let $\mathbf{\Omega}$ be a non-empty set. Show that the following are $\boldsymbol{\delta}$-fields.

i. $\{ \mathbf{\Omega} , \Phi\}$ (3 marks)

ii. Power set, $\mathbf{P}\{ \mathbf{\Omega} \}$ (3 marks)

iii. $\boldsymbol{\pi}_1 \cap \boldsymbol{\pi}_2$ where $\boldsymbol{\pi}_1$ and $\boldsymbol{\pi}_2$ are $\boldsymbol{\delta}$-fields defined on $\mathbf{\Omega}$ **.** (4 marks)

d) Let $\{ \mathbf{\Omega} , \boldsymbol{\pi} \}$ be a measurable space. Suppose $\mathbf{A}_1, \mathbf{A}_2 \in \boldsymbol{\pi}$ and $\boldsymbol{\mu}$ is a measure on $\boldsymbol{\pi}$**.** Show that $\boldsymbol{\mu}(\mathbf{A}_1 \cup \mathbf{A}_2) \leq \boldsymbol{\mu}(\mathbf{A}_1) + \boldsymbol{\mu} (\mathbf{A}_2)$. (5 marks)

QUESTION TWO

a) State the axioms of ;

i. a measure, **μ**

ii. a probability measure, **P** (6 marks)

b) Let x_n be a sequence of random variables. State and prove the Chebychevs inequality. (6 marks)

c) Define;

i. Convergence in probability

ii. Convergence with probability 1

iii. Convergence in mean square

iv. Convergence in distribution (8 marks)

QUESTION THREE

a) Let { **Ω** , **π, P** } be a probability measure and $\mathbf{P}_B$ be defined by;

$$\mathbf{P}_B(A)=\mathbf{P}(A/B)=\mathbf{P}(A \cap B)/\mathbf{P}(B) \quad ; P(B)>0.$$

Show that $\mathbf{P}_B$ is a probability measure. (7 marks)

b) Show that an indicator function is a measure function (3 marks)

d) Let **Ω={2}.**

i. Determine the power set, **P**{ Ω} (1 mark)

ii. Show that **P**{ Ω}obtained in (a) above is a **δ**-algebra. (4 marks)

e) Let $\mathbf{x}_1,\ \mathbf{x}_{2,} \ldots\ldots, \mathbf{x}_n$ be a sequence of random variables. Show that convergence in mean square implies convergence in probability. (5 marks)

QUESTION FOUR

a) State the following theorems;

i. Radon-Nikodym theorem (3 marks)

ii. Fubini's theorem (5 marks)

b) State the axioms of a **δ**-field and hence show that a **δ**-algebra is a **δ**-field. (10 marks)

c) Define a Stieltjes measure. (2 marks)

QUESTION FIVE

a) State and prove the continuity theorem of a measure, **μ** (10 marks)

b) Define the power set of a set, **P**{ Ω}and hence or otherwise

i. determine the power set of a set {1,2}.

ii. Determine whether the power set in (i) above is a is a **δ**-algebra. (10 marks)