MACHAKOS UNIVERSITY

University Examinations 2020/2021 Academic Year

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF PHYSICAL SCIENCES

THIRD YEAR FIRST SEMESTER EXAMINATION FOR

BACHELOR OF EDUCATION (SCIENCE)

SPH 301: QUANTUM MECHANICS 1

DATE: 10/8/2021 **TIME: 8.30-10.30 AM**

INSTRUCTIONS:

The paper consists of **two** sections.

Section **A** is **compulsory** (30 marks).

Answer any **two** questions from section **B** (each 20 marks).

SECTION A

QUESTION ONE (30 MARKS)

a) State the expectation value for position, momentum and energy for a normalized wave function. (3 marks)

b) Explain three failures of classical mechanics from the photoelectric experiment that led to quantum mechanics approach (4 marks)

c) Show that for a one-dimensional square - integrable wave-packet,

$$\int_{-\infty}^{\infty} j(x)dx = \frac{\langle p \rangle}{m}$$

Where$j(x)$, is the probability current? (5 marks)

d) Show that $[x, p_y]$ commute (5 marks)

e) A particle of mass m is trapped in a one dimensional box with a potential described by

$$V(x) = \begin{cases} 0 & 0 \le x \le a \\ \infty & \text{Otherwise} \end{cases}$$

Solve the Schrödinger equation for this potential. Taking $\Psi(x,t) = \Psi(x)e^{-iEt/\hbar}$ (8 marks)

f) i. State the mathematical operator for position, momentum and energy for a normalised wave function (3 marks)

ii. Write down expression for time independent Schrodinger equation in one dimension. (2 marks)

QUESTION TWO (20 MARKS)

a) State two properties of Schrödinger equation (2 marks)

b) Derive Time – dependent Schrödinger equation and hence write it in three dimension given that the wave equation is $\Psi = Ae^{i(kx-\omega t+\Phi)}$ (18 marks)

QUESTION THREE (20 MARKS)

a) What are the two conditions for normalization of a wave function (2 marks)

b) Consider a particle described by a wave function *ρ(r, t)*. Calculate the time-derivative $\frac{\partial\rho(r,t)}{\partial t}$ where ρ*(r, t)* is the probability density and show that the continuity equation $\frac{\partial\rho(r,t)}{\partial t} + \nabla.\vec{\mathrm{J}}(\mathrm{r,t}) = 0$ is valid, where ***J****(r,t)* is the probability current (18 marks)

QUESTION FOUR (20 MARKS)

Prove that $d\langle x\rangle dt = \langle p\rangle m$ Where ⟨x⟩and ⟨p⟩are the mean values of the coordinate and momentum of the particle, respectively.

QUESTION FIVE (20 MARKS)

a) State the condition for an operator to be hermitian (2 marks)

b) Show that the momentum operator -i∂x is Hermitian operator. Obtain eigenfunction of momentum operator (18 marks)