MACHAKOS UNIVERSITY

University Examinations 2018/2019

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS, STATISTICS AND ACTUARIAL SCIENCE

FIRST YEAR SECOND SEMESTER SPECIAL/ SUPPLEMENTARY EXAMINATION FOR BACHELOR OF SCIENCE (ELECTRICAL AND ELECTRONICS)
BACHELOR OF SCIENCE (MECHANICAL ENGINEERING)
BACHELOR OF CIVIL ENGINEERING

ECU106-ENGINEERING MATHEMATICS III

DATE: 26/9/2019 **TIME: 8:30 – 10:30 AM**

INSTRUCTIONS:

Attempt ***question one (compulsory)*** and ***any other two*** questions.

QUESTION ONE (COMPULSORY) (30MARKS)

a) Determine the value of the constant a such that the vectors $2i - j + k,\ i + 2j - 3k,\ \ and\ \ 3i + aj + 5k$ are coplanar (3 marks)

b) Let $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 1 \\ 4 & 1 & 2 \end{bmatrix}$. Find the determinant of the matrix A. (3 marks)

c) Find the equation of the plane determined by the points $A(2,-1,1),\ B(3,2,-1)$ and $C(-1,3,2)$. Also find the coordinates of the point where this line meets the plane $z = 0$ (4 marks)

d) If $\vec{A} = t^2\,\hat{i} - t\,\hat{j} + (2t-1)\hat{k},\ and\ \vec{B} = (2t-3)\hat{i} + \hat{j} - t\,\hat{k}$, evaluate $\frac{d}{dt}\left[\vec{A}\times\frac{d\vec{B}}{dt}\right]$

(4 marks)

e) Use Gramer's rule to solve

$$2x+3y-z=1$$
$$3x+5y+2z=8$$
$$x-2y-3z=-1$$

(5 marks)

f) Prove that the diagonals of a parallelogram bisect each other (5 marks)

g) Express $w=(-7,7,11)$ as a linear combination of the vectors $v_1=(1,2,1)$, $v_2=(-4,-1,2)$ and $v_3=(-3,1,3)$ (6 marks)

QUESTION TWO (20 MARKS)

a) Find the volume of the parallelepiped whose edges are represented by

$\vec{A} = 2\hat{i} - 3\hat{j} + 4\hat{k}\ ,\vec{B} = \hat{i} + 2\hat{j} - \hat{k}\ and\ \vec{C} = 3\hat{i} - \hat{j} + 2\hat{k}$

(5 marks)

b) Find the inverse of the following matrices

$$A=\begin{bmatrix}4 & 0 & 1\\ 2 & 2 & 0\\ 3 & 1 & 1\end{bmatrix}$$

(7 marks)

c) Determine the equation of the plane through $(-1,2,3)$ and perpendicular to the planes

$$2x-3y+4z=1$$
$$3x-5y+2z=3$$

(8 marks)

QUESTION THREE (20 MARKS)

a) Transpose the following matrix

$$\begin{bmatrix}5 & -7 & 3\\ 4 & 2 & 0\\ 3 & 4 & 7\end{bmatrix}$$

(3 marks)

b) Given that $A = 5\hat{i} - 2\hat{j} + 3\hat{k},\ \vec{B} = 3\hat{i} + \hat{j} - 2\hat{k}\ \ and\ \vec{C} = \hat{i} - 3\hat{j} + 4\hat{k}$. Determine $\vec{A}\cdot\left(\vec{B}\times\vec{C}\right)$

(5 marks)

c) Find the acute angle between the lines

$\frac{x-1}{5}=\frac{y+2}{4}=\frac{z}{6}$ and $\frac{x+3}{3}=\frac{y-2}{7}=\frac{z+1}{-1}$ (5 marks)

d) If $B=|b_{ij}|$ is obtained from $A=|a_{ij}|$ by adding to each element of the r^{th} row (column) of A a constant C times the corresponding element of the s^{th} row (column) $r \neq s$ of A, then $\det B = \det A$. Prove. (7 marks)

QUESTION FOUR (20 MARKS)

a) Let $A = \begin{bmatrix} 1 & 2 & 1 \\ 3 & 1 & 0 \\ 2 & 1 & 2 \end{bmatrix}$.

Compute

i. $|A|$ (2 marks)

ii. $(adjA)A$ and $A(adjA)$ (3 marks)

iii. $adjA$ (5 marks)

b) Calculate the value of k for the vectors $u = (1, k)$ and $v = (-4, k)$ knowing that they are orthogonal.
(5 marks)

c) Find the rank of the following matrix

$$\begin{bmatrix} 2 & 4 & 3 \\ 3 & -3 & 2 \\ 2 & 0 & 3 \end{bmatrix}$$ (5 marks)

QUESTION FIVE (20 MARKS)

a) Find the angle between the given two vectors $a = i + 4j + 8k$ and $b = 5i + 4j + 3k$
(5 marks)

b) Let $A = \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$ find $(A+B)^2 - (A^2 + 2AB + B^2)$ (5 marks)

c) Solve by inverse matrix method

$$x_1 + 3x_2 + 2x_3 = 3$$
$$2x_1 + 4x_2 + 2x_3 = 8$$
$$x_1 + 2x_2 - x_3 = 10$$
(10 marks)