MACHAKOS UNIVERSITY

University Examinations for 2020/2021 Academic Year

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

FOURTH YEAR FIRST SEMESTER EXAMINATION FOR

BACHELOR OF SCIENCE (MATHEMATICS AND COMPUTER SCIENCE)

SMA 434: GAS DYNAMICS

DATE: 16/8/2021 **TIME: 8:30 – 10:30 AM**

INSTRUCTIONS

Answer question **one** (**Compulsory**) and any other **two** questions.

QUESTION ONE (COMPULSORY) (30 MARKS)

a) Describe how aerodynamics aids in the flying of airplanes (2 marks)

b) A calorically perfect ideal gas is undergoing an isentropic process. What two factors can you consider to justify that the gas is indeed undergoing the process. (2 marks)

c) Compute the density of air at 9600 Nm^{-2}, 480 K.

Take the gas constant $R=53.3Jkg^{-1}K^{-1}$ (3 marks)

d) Consider a column vector $h_i(i = 1,2,3, \ldots, n)$ of n variables, considering two functions $f_i(h_i)$ and $g_i(h_i)$, where all variables are functions of x and t, Formulate the necessary system of partial differential equation for the system to be in conservative form. (3 marks)

e) Consider a fluid flowing between a slot separated by two plates; the lower at $y = 0$ is stationery and the upper at $y = h$ is moving with a velocity U. Given that the velocity is only in the x direction and only a function of y, the flow has no imposed pressure gradient or body force and the flow has no acceleration, assuming constant viscosity and using the linear momentum principle.

(i) Show that the flow is linear. (3 marks)

(ii) Suppose that the viscosity of the fluid is 0.30 $kgm^{-1}s^{-1}$. Compute the shear stress τ in the fluid if it is moving with a velocity, U = 5 ms^{-1} and the clearance h between the plates is 2.5 cm. (2 marks)

f) Consider a gas flowing through a stream tube. Given that at one section of the tube, the flow has a pressure of 200 Nm^{-2}, a temperature of 300 K and is moving with a velocity of 250 ms^{-1}, Compute the temperature at another section of the tube where the pressure reduces to 40 Nm^{-2} and the speed increases to 1100 ms^{-1}. *Take the specific heat of the gas at constant pressure,* $c_p = 2295\ Jkg^{-1}K^{-1}$. (4 marks)

g) Hydrogen gas has a static temperature of 298 K and stagnation temperature of 523 K. Calculate its Mach number and hence classify the disturbance as either subsonic or supersonic. *Take the ratio of specific heats to be* $\gamma = 1.41$. (3 marks)

h) Consider air at a temperature of 600 K flowing through a converging-diverging nozzle which has a cross sectional area of 1 cm^2. Given that the initial mass flow rate is 0.04125 kgs^{-1}, the initial and exit pressures are 200 kPa and 191.5 kPa respectively. Compute;

i) Mass flow rate at the exit, (3 marks)

ii) Exit velocity. (5 marks)

Take $\gamma = 1.4$, $R = 287\ Jkg^{-1}K^{-1}$ *and* $c_p = 1004.5\ Jkg^{-1}K^{-1}$.

QUESTION TWO (20 MARKS)

a) Consider a flow between a slot separated by two plates; the lower at $y = 0$ is stationery and the upper at $y = h$ is moving with a velocity U. Given that the velocity is only in the x direction and only a function of y, the flow is driven by a pressure difference in the x direction such that at $x = 0, P = P_o$ and at $x = l, P = P_1$. If there is no body force, the flow has no acceleration, assuming constant viscosity and using the linear momentum principle, Compute the velocity profile parameterized by P_o, P, h, U and μ. 10 marks

b) Considering an ideal gas where pressure P is a function of temperature, T and Volume, V given by $P(T, V) = \frac{RT}{V}$. Using the laws of thermodynamics, derive the relation for;

i) The gas constant $R = c_p - c_v$ (3 marks)

ii) The total internal energy $e(T) = e_o + c_v(T - T_o)$ (4 marks)

c) Given that the speed of sound in a general material, $c(T, \rho) = \sqrt{\left(\frac{dp}{d\rho}\right)_T + \frac{T}{c_v\rho^2}\left[\left(\frac{dP}{dT}\right)_\rho\right]^2}$

Simplify this expression and prove that for an ideal gas, the speed is only a function of temperature and given by $c(T) = \sqrt{\gamma RT}$ (3 marks)

QUESTION THREE (20 MARKS)

a) Consider a calorically perfect ideal gas flowing through a duct of length 150m having a uniform cross section of area 0.03 m^2. Given that at one end of the duct, the gas has a pressure of 150 kPa, a temperature of 300 K and is flowing at a velocity of 12 ms^{-1}. If the gas is isobarically heated with a constant heat flux along the entire surface of the duct such that at the other end it has a pressure of 150 kPa and a temperature of 500K, Calculate;

i) The mass flow rate, (4 marks)

ii) Velocity of the gas at the other end. (4 marks)

Take R=287 $Jkg^{-1}K^{-1}$

b) Consider an airplane flying through still air at a velocity of 200 ms^{-1}. Assuming steady isentropic flow of a calorically perfect ideal gas, given that the ambient air has a temperature of 288 K and a pressure of 101.3 kPa, Compute;

i) The Mach number and hence classify the disturbance, (4 marks)

ii) Temperature, pressure and density at the nose of the airplane. (8 marks)

Take $\gamma=\frac{7}{5}$ *and* $R=287\ Jkg^{-1}K^{-1}$

QUESTION FOUR (20 MARKS)

Consider air flow through a normal shock wave. Given that the upstream conditions are $u_1 = 600ms^{-1}, T_{01} = 500K, P_{01} = 700kPa$, using Rankine-Hugoniot relations Calculate the downstream conditions M_2, u_2, T_2 and P_{02} .*Take* $\gamma=1.4$, $c_p = 1000Jkg^{-1}K^{-1}$ *and* $R=287\ Jkg^{-1}K^{-1}$

QUESTION FIVE (20 MARKS)

a) Consider a gas flowing over a wedge inclined at an angle of 20^0 to the horizontal. For a calorically perfect ideal gas, if the upstream conditions are $P_1 = 100kPa, T_1 = 300K$ and $M_1 = 3$. Using Hodograph and characteristic relations, compute the downstream pressure and temperature for a shock angle of;

i) $\beta = 37.76^0$ relating to a weak oblique shock wave, (5 marks)

ii) $\beta = 82.15^0$ relating to a strong oblique shock wave, (5 marks)

iii) $\beta = -9.91^0$ relating to a rarefaction shock. (5 marks)

b) Consider a calorically perfect ideal gas. Given that it has a temperature of 300 K and is moving with a velocity of 500 ms^{-1}, compute the Prandtl-Mayer function $\nu(M)$.

$Take\ \gamma = 1.4\ and\ R = 287\ Jkg^{-1}K^{-1}$ (5 marks)