



MACHAKOS UNIVERSITY

University Examinations 2016/2017

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

FIRST YEAR SECOND SEMESTER EXAMINATION FOR THE DEGREE OF
BACHELOR OF ARTS (MATHEMATICS AND ECONOMICS)

SMA 103: ANALYTICAL GEOMETRY

DATE: 2/6/2017

TIME: 2:00 – 4:00 PM

INSTRUCTION:

Answer Question ONE which is compulsory and any other TWO Questions

QUESTION ONE 30 MARKS

- a) Write down the equation of a straight line through (5,11) parallel to the x-axis (4 marks)
- b) Prove that $\frac{1+\cot \theta}{1+\tan \theta} = \cot \theta$ (4 marks)
- c) b) Show that part of the line $4y - 3x - 9 = 0$ is chord of the circle $x^2 + y^2 - 2x - 6y - 15 = 0$ and find the length of this chord. (5 marks)
- d) Find the distance from point P(-2, -3) to the line $8x + 15y - 24 = 0$. (4 marks)
- e) Determine the radius and the coordinates of the circle given by the equation $x^2 + y^2 - 8x - 2y + 8 = 0$ (5 marks)
- f) Express & define the polar equation $r^2(b^2 \cos^2 \phi + a^2 \sin^2 \phi) = a^2 b^2$ in Cartesian form. (3 marks)
- g) Find the equation of a parabola whose focus is (2,0) and the directrix is $x = 2$ (5 marks)

QUESTION TWO (20 MARKS)

- a) Find the diretrix, eccentricity & foci of the ellipse $4x^2 + 9y^2 = 36$. Hence sketch a graph (8 marks)
- b) The straight line $x - y - 6 = 0$ cuts the curve $y^2 = 8x$ at P and Q. calculate the length of PQ (6 marks)
- c) Show that the equation $y = 5x - 2x^2$ represents a parabola and state the distance between its directrix and focus (6 marks)

QUESTION THREE (20 MARKS)

- a) Find the equation of the tangent and normal to the circle $x^2 + y^2 + 4x - 10y = -4$ at the point (2, 8) on the circumference of the circle (6 marks)
- b) Sketch the curves given by the following polar equations
- (i) $r = 1 + \cos\theta$
- (ii) $r = a\cos 2\theta$, $0^\circ \leq \theta \leq 360^\circ$ (6 marks)
- c) Find the equation of the bisectors of the angle between $4x - 3y + 1 = 0$ and $3x - 4y + 3 = 0$ (4 marks)
- d) Prove that $\tan 2\theta = \frac{2\tan \theta}{1 - \cos^2 \theta}$ (4 marks)

QUESTION FOUR (20 MARKS)

- a) Find the coordinate of the foci, the eccentricity, the length of the latus rectum and the direction of the hyperbola $\frac{x^2}{36} - \frac{y^2}{4} = 1$ (8 marks)
- b) Plot the graph of the curve given parametrically as $x = t^2 - 4$, $y = t^3 - 4t$, $-3 \leq t \leq 3$ (6 marks)
- c) Show that the equation of a parabola centre (0, q) and directrix $y = -q$ is given by $x^2 = 4qy$ (6 marks)

QUESTION FIVE (20 MARKS)

- a) Find the angle between the lines $y = 2x + 5$ and $y = 12x - 1$ (2 marks)
- b) Find the centre, vertices and the asymptotes of the hyperbola with the equation $4x^2 - 5y^2 + 40x - 30y - 45 = 0$ (10 marks)
- c) Sketch the region in the xy - plane whose coordinates satisfy the inequalities $x^2 + 4y^2 \geq 1$ and $4x^2 + 9y^2 = 36$ (8 marks)