



MACHAKOS UNIVERSITY

University Examinations 2020/2021 Academic Year

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

FIRST YEAR SPECIAL /SUPPLEMENTARY EXAMINATION FOR

BACHELOR OF SCIENCE (ELECTRICAL AND ELECTRONICS ENGINEERING)

BACHELOR OF SCIENCE IN (MECHANICAL ENGINEERING)

BACHELOR OF SCIENCE IN (CIVIL ENGINEERING)

ECU 104: ENGINEERING MATHEMATICS 1

DATE: 29/3/2021

TIME: 11.00-1.00 PM

INSTRUCTIONS

Answer question ONE and any other TWO questions

QUESTION ONE (30 MARKS)

- a) The angle between two lines is 45° . If the slope m_1 of l_1 is $\frac{2}{3}$. Determine the slope of l_2 . (3 marks)
- b) Find the equation of the circle which has the points $(0, -1)$ and $(2, 3)$ as ends of a diameter. (5 marks)
- c) Find the equation of the tangent at the point of contact $(8, -3)$ of the circle whose equation is $x^2 + y^2 - 6x + 4y - 12 = 0$. (5 marks)
- d) Prove that the circles $x^2 + y^2 - 16x - 12y + 75 = 0$ and $5x^2 + 5y^2 - 32x - 24y + 75 = 0$ touch each other. (5 marks)
- e) Determine the length of the tangent drawn from the point $(5, 7)$ to the circle $x^2 + y^2 - 4x - 6y + 9 = 0$. (5 marks)
- f) Derive the equation of the parabola with its vertex at $(3, 2)$ and its focus at $(5, 2)$. (4 marks)
- g) Convert the polar equation $r = 4 \sin \theta$ to rectangular form. (3 marks)

QUESTION TWO (20 MARKS)

- a) Show that the point $(1,1)$ is equidistant from the lines $3x + 4y = 12$,
 $5x - 12y + 20 = 0$ and $4x - 3y = 6$. (6 marks)
- b) Find the equations of the tangent to the curve $xy = 6$ which are parallel to the curve $3y + 2x = 0$ (6 marks)
- c) A circle passes through the points $A(-5,2)$, $B(-3,-4)$ and $C(1,8)$. Find the point of intersection of the perpendicular bisectors of AB and BC. What is the equation of the circle? (8 marks)

QUESTION THREE (20 MARKS)

- a) Find the equations to the tangents and normal to the parabola $y^2 = 16x$ at points $(16,16)$ and $(1, -4)$. The tangents intersect at the point T and normal intersect at R. Prove that the line TR is parallel to the axis of the parabola. (10 marks)
- b) Find the vertex, focus, equation of the directrix and the length of latus rectum if the equation of the parabola is $3x^2 - 9x - 5y - 2 = 0$. Hence sketch the curve. (10 marks)

QUESTION FOUR (20 MARKS)

- a) (i) For the vectors $a = i + 4j - 2k$ and $b = 2i - j + 3k$ find $a \times b$. (3 marks)
- (ii) Determine the angle between the vectors a and b when $a = i + 2j - 3k$ and $b = 2i - j + 4k$. (4 marks)
- b) A force $F = 10i + 2j - k$ newtons displaces an object from $A = i + j + k$ to $B = 2i - j + 3k$ (in metres). Find the work done. (5 marks)
- c) Find the moment and magnitude of the moment of force of $(i + 2j - 3k)$ newtons about point b having coordinates $(0,1,1)$ when the forces acts on a line through A whose coordinates are $(1,3,4)$. (8 marks)

QUESTION FIVE (20 MARKS)

- a) Find the coordinates of the centre, vertices and foci, the equations of the directrices, the equations of the asymptotes and sketch the graph of the hyperbola whose equation is $9x^2 - 16y^2 - 18x - 64y - 199 = 0$. (10 marks)
- b) Find the equations of the tangent and normal to the ellipse $5x^2 + 3y^2 = 137$, at the point in the first quadrant whose y coordinate is 2. (6 marks)

- c) Derive the equation of the ellipse having its centre at the origin, one focus at $(0,3)$ and the length of semi-major axis 5. (4 marks)