



MACHAKOS UNIVERSITY

University Examinations 2017/2018

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

SECOND YEAR FIRST SEMESTER EXAMINATIONS FOR
BACHELOR OF SCIENCE IN MATHEMATICS AND COMPUTER SCIENCE

BACHELOR OF SCIENCE IN STATISTICS AND PROGRAMMING

BACHELOR OF SCIENCE IN MATHEMATICS

BACHELOR OF EDUCATION (ARTS AND SCIENCE)

BACHELOR OF ECONOMICS AND STATISTICS

BACHELOR OF ARTS

SMA 260 - PROBABILITY AND STATISTICS I

DATE: 20/12/2017

TIME: 8:30 – 10:30 AM

INSTRUCTIONS

1. Answer **Question 1** and any other **two** questions.
2. You must have the following items for this paper:
 - Scientific Calculator;
 - Statistical tables

1. (a) Outline **four** characteristics of the normal probability distribution. (2 marks)
- (b) A discrete random variable x has a probability function given by:

x	0	1	2	3	4	5
$f(x)$	0.10	0.15	0.30	0.25	0.15	0.05

- (i) Show that the function $f(x)$ is a probability mass function (p.m.f.). (2 marks)
- (ii) Determine the mean and the variance of the random variable x in the probability distribution. (6 marks)
- (iii) Determine the probability that x is at least 3. (2 marks)

- (c) A discrete random variable x has a Poisson probability distribution with probability mass function (p.m.f.) given by:

$$f(x) = \begin{cases} \frac{\lambda^x e^{-\lambda}}{x!} & \text{for } x = 0, 1, 2, 3, \dots, n. \\ 0 & \text{; elsewhere} \end{cases}$$

Prove that the mean of x and the variance of x are both equal to λ (6 marks)

- (d) A continuous random variable x is given by the function

$$f(x) = \begin{cases} \frac{1}{2}x & \text{for } 0 \leq x \leq 2. \\ 0 & \text{elsewhere} \end{cases}$$

- (i) Show that the function $f(x)$ is a probability density function (p.d.f.).
(ii) Hence determine the probability: $P\left(\frac{1}{2} \leq x \leq 1\right)$. (4 marks)

- (e) A continuous random variable x is given by the probability density function

$$f(x) = \begin{cases} \frac{1}{2}(x+1) & \text{for } -1 \leq x \leq 1. \\ 0 & \text{elsewhere} \end{cases}$$

Determine the variance of $(5x + 10)$ (4 marks)

- (f) A continuous random variable x is normally distributed with parameters mean $\mu = 10$ and variance $\sigma^2 = 36$.
Determine the probability: $P(4 \leq x \leq 6)$. (4 marks)

2. A continuous random variable x is given by the function

$$f(x) = \begin{cases} \frac{2}{9}(9x - x^2 - 18) & \text{for } 3 \leq x \leq 6. \\ 0 & \text{elsewhere} \end{cases}$$

- (a) Show that the function $f(x)$ is a probability density function (p.d.f.). (3 marks)
(b) Determine the following measures about the random variable x in the probability distribution:
(i) the mode of x ;
(ii) the mean of x ;
(iii) the variance of x . (13 marks)
(c) Determine the following probability from the distribution:
 $P(2 \leq x \leq 5)$. (4 marks)

3. (a) The National Transport and Safety Authority claims that only 60% of the drivers on Kenyan roads were trained in driving schools. A random sample of 12 drivers on the road was taken from Kenya. Determine the probability that among these drivers the following were trained in a driving school:
- (i) exactly 5 drivers;
 - (ii) at least 4 drivers;
 - (iii) Between 4 and 6 drivers inclusive. (10 marks)
- (b) The daily water usage of residents in Thika Town has been found to be normally distributed with a mean of 20 gallons and a standard deviation of 5 gallons:
- (i) Determine the probability that a resident selected at random will use less than 15 gallons of water per day; (3 marks)
 - (ii) Determine the probability that a resident selected at random will use more than 30 gallons of water per day; (3 marks)
 - (iii) Determine the percentage proportion of Thika residents who use between 25 and 30 gallons per day. (4 marks)
4. (a) Outline **three** characteristics of the binomial probability distribution. (3 marks)
- (b) The number of people who become ill each year from eating a kind of poisonous plant in a certain region has been found to have a Poisson probability distribution with rate parameter $\lambda = 1.6$. Determine the probability of obtaining at random the following number of people who have fallen ill from the plant in a given year:
- (i) exactly 2 people;
 - (ii) at least 7 people; (7 marks)
- (d) The operational lifespan of a given brand of a photocopying machines has been found to be normally distributed with a mean of 4.8 years and a standard deviation of 1.6 years.
- (i) Determine the proportion of these photocopying machines that will have a lifespan of between 3.8 years and 6.6 years; (4 marks)
 - (ii) If these photocopying machines have a warranty period of 2 years, determine the proportion of original sales which will require replacement through this warranty. (3 marks)
 - (iii) If the manufacturer of these photocopying machines wants only 5% of the machines to be replaced through this warranty, determine the warranty period that should be set to achieve this. (3 marks)

5. (a) A continuous random variable x has a probability density function given by:

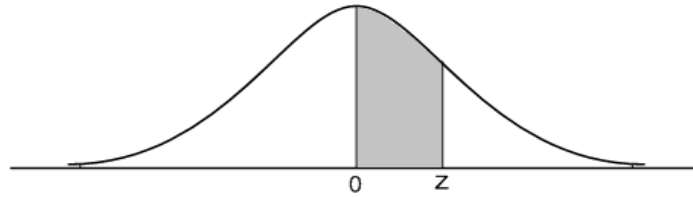
$$f(x) = \begin{cases} x e^{-x} ; & \text{for } x > 0 \\ 0 & \text{elsewhere} \end{cases}$$

- (i) Determine the moment generating function of x .
(ii) Hence determine the mean and the variance of x : (10 marks)

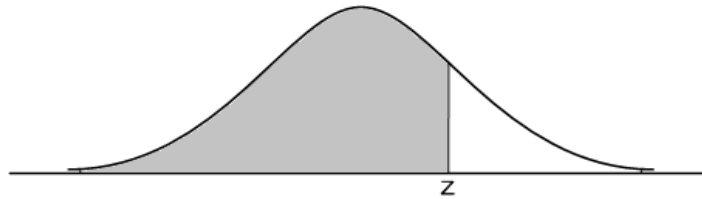
- (b) A continuous random variable x has a probability density function given by:

$$f(x) = \begin{cases} 2x ; & \text{for } 0 \leq x \leq 1. \\ 0 & \text{elsewhere} \end{cases}$$

- (i) Determine the probability density function of a continuous random variable $y = 8x^3$ using the change of variable technique. (5 marks)
(ii) Hence determine the following:
I. the mean of y ;
II. the probability $P(Y > 4)$; (5 marks)



Area under the standard normal curve from 0 to Z										
Z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.500000	0.503989	0.507978	0.511966	0.515953	0.519939	0.523922	0.527903	0.531881	0.535856
0.1	0.539828	0.543795	0.547758	0.551717	0.555670	0.559618	0.563559	0.567495	0.571424	0.575345
0.2	0.579260	0.583166	0.587064	0.590954	0.594835	0.598706	0.602568	0.606420	0.610261	0.614092
0.3	0.617911	0.621720	0.625516	0.629300	0.633072	0.636831	0.640576	0.644309	0.648027	0.651732
0.4	0.655422	0.659097	0.662757	0.666402	0.670031	0.673645	0.677242	0.680822	0.684386	0.687933
0.5	0.691462	0.694974	0.698468	0.701944	0.705401	0.708840	0.712260	0.715661	0.719043	0.722405
0.6	0.725747	0.729069	0.732371	0.735653	0.738914	0.742154	0.745373	0.748571	0.751748	0.754903
0.7	0.758036	0.761148	0.764238	0.767305	0.770350	0.773373	0.776373	0.779350	0.782305	0.785236
0.8	0.788145	0.791030	0.793892	0.796731	0.799546	0.802337	0.805105	0.807850	0.810570	0.813267
0.9	0.815940	0.818589	0.821214	0.823814	0.826391	0.828944	0.831472	0.833977	0.836457	0.838913
1.0	0.841345	0.843752	0.846136	0.848495	0.850830	0.853141	0.855428	0.857690	0.859929	0.862143
1.1	0.864334	0.866500	0.868643	0.870762	0.872857	0.874928	0.876976	0.879000	0.881000	0.882977
1.2	0.884930	0.886861	0.888768	0.890651	0.892512	0.894350	0.896165	0.897958	0.899727	0.901475
1.3	0.903200	0.904902	0.906582	0.908241	0.909877	0.911492	0.913085	0.914657	0.916207	0.917736
1.4	0.919243	0.920730	0.922196	0.923641	0.925066	0.926471	0.927855	0.929219	0.930563	0.931888
1.5	0.933193	0.934478	0.935745	0.936992	0.938220	0.939429	0.940620	0.941792	0.942947	0.944083
1.6	0.945201	0.946301	0.947384	0.948449	0.949497	0.950529	0.951543	0.952540	0.953521	0.954486
1.7	0.955435	0.956367	0.957284	0.958185	0.959070	0.959941	0.960796	0.961636	0.962462	0.963273
1.8	0.964070	0.964852	0.965620	0.966375	0.967116	0.967843	0.968557	0.969258	0.969946	0.970621
1.9	0.971283	0.971933	0.972571	0.973197	0.973810	0.974412	0.975002	0.975581	0.976148	0.976705
2.0	0.977250	0.977784	0.978308	0.978822	0.979325	0.979818	0.980301	0.980774	0.981237	0.981691
2.1	0.982136	0.982571	0.982997	0.983414	0.983823	0.984222	0.984614	0.984997	0.985371	0.985738
2.2	0.986097	0.986447	0.986791	0.987126	0.987455	0.987776	0.988089	0.988396	0.988696	0.988989
2.3	0.989276	0.989556	0.989830	0.990097	0.990358	0.990613	0.990863	0.991106	0.991344	0.991576
2.4	0.991802	0.992024	0.992240	0.992451	0.992656	0.992857	0.993053	0.993244	0.993431	0.993613
2.5	0.993790	0.993963	0.994132	0.994297	0.994457	0.994614	0.994766	0.994915	0.995060	0.995201
2.6	0.995339	0.995473	0.995604	0.995731	0.995855	0.995975	0.996093	0.996207	0.996319	0.996427
2.7	0.996533	0.996636	0.996736	0.996833	0.996928	0.997020	0.997110	0.997197	0.997282	0.997365
2.8	0.997445	0.997523	0.997599	0.997673	0.997744	0.997814	0.997882	0.997948	0.998012	0.998074
2.9	0.998134	0.998193	0.998250	0.998305	0.998359	0.998411	0.998462	0.998511	0.998559	0.998605
3.0	0.998650	0.998694	0.998736	0.998777	0.998817	0.998856	0.998893	0.998930	0.998965	0.998999
3.1	0.999032	0.999065	0.999096	0.999126	0.999155	0.999184	0.999211	0.999238	0.999264	0.999289
3.2	0.999313	0.999336	0.999359	0.999381	0.999402	0.999423	0.999443	0.999462	0.999481	0.999499
3.3	0.999517	0.999534	0.999550	0.999566	0.999581	0.999596	0.999610	0.999624	0.999638	0.999651
3.4	0.999663	0.999675	0.999687	0.999698	0.999709	0.999720	0.999730	0.999740	0.999749	0.999758
3.5	0.999767	0.999776	0.999784	0.999792	0.999800	0.999807	0.999815	0.999822	0.999828	0.999835
3.6	0.999841	0.999847	0.999853	0.999858	0.999864	0.999869	0.999874	0.999879	0.999883	0.999888
3.7	0.999892	0.999896	0.999900	0.999904	0.999908	0.999912	0.999915	0.999918	0.999922	0.999925
3.8	0.999928	0.999931	0.999933	0.999936	0.999938	0.999941	0.999943	0.999946	0.999948	0.999950
3.9	0.999952	0.999954	0.999956	0.999958	0.999959	0.999961	0.999963	0.999964	0.999966	0.999967
4.0	0.999968	0.999970	0.999971	0.999972	0.999973	0.999974	0.999975	0.999976	0.999977	0.999978
4.1	0.999979	0.999980	0.999981	0.999982	0.999983	0.999983	0.999984	0.999985	0.999985	0.999986
4.2	0.999987	0.999987	0.999988	0.999988	0.999989	0.999989	0.999990	0.999990	0.999991	0.999991
4.3	0.999991	0.999992	0.999992	0.999993	0.999993	0.999993	0.999993	0.999994	0.999994	0.999994
4.4	0.999995	0.999995	0.999995	0.999995	0.999996	0.999996	0.999996	0.999996	0.999996	0.999996
4.5	0.999997	0.999997	0.999997	0.999997	0.999997	0.999997	0.999997	0.999998	0.999998	0.999998
4.6	0.999998	0.999998	0.999998	0.999998	0.999998	0.999998	0.999998	0.999998	0.999999	0.999999
4.7	0.999999	0.999999	0.999999	0.999999	0.999999	0.999999	0.999999	0.999999	0.999999	0.999999
4.8	0.999999	0.999999	0.999999	0.999999	0.999999	0.999999	0.999999	0.999999	0.999999	0.999999
4.9	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000
5.0	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000



Area under the standard normal curve from 0 to Z										
Z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.000000	0.003989	0.007978	0.011966	0.015953	0.019939	0.023922	0.027903	0.031881	0.035856
0.1	0.039828	0.043795	0.047758	0.051717	0.055670	0.059618	0.063559	0.067495	0.071424	0.075345
0.2	0.079260	0.083166	0.087064	0.090954	0.094835	0.098706	0.102568	0.106420	0.110261	0.114092
0.3	0.117911	0.121720	0.125516	0.129300	0.133072	0.136831	0.140576	0.144309	0.148027	0.151732
0.4	0.155422	0.159097	0.162757	0.166402	0.170031	0.173645	0.177242	0.180822	0.184386	0.187933
0.5	0.191462	0.194974	0.198468	0.201944	0.205401	0.208840	0.212260	0.215661	0.219043	0.222405
0.6	0.225747	0.229069	0.232371	0.235653	0.238914	0.242154	0.245373	0.248571	0.251748	0.254903
0.7	0.258036	0.261148	0.264238	0.267305	0.270350	0.273373	0.276373	0.279350	0.282305	0.285236
0.8	0.288145	0.291030	0.293892	0.296731	0.299546	0.302337	0.305105	0.307850	0.310570	0.313267
0.9	0.315940	0.318589	0.321214	0.323814	0.326391	0.328944	0.331472	0.333977	0.336457	0.338913
1.0	0.341345	0.343752	0.346136	0.348495	0.350830	0.353141	0.355428	0.357690	0.359929	0.362143
1.1	0.364334	0.366500	0.368643	0.370762	0.372857	0.374928	0.376976	0.379000	0.381000	0.382977
1.2	0.384930	0.386861	0.388768	0.390651	0.392512	0.394350	0.396165	0.397958	0.399727	0.401475
1.3	0.403200	0.404902	0.406582	0.408241	0.409877	0.411492	0.413085	0.414657	0.416207	0.417736
1.4	0.419243	0.420730	0.422196	0.423641	0.425066	0.426471	0.427855	0.429219	0.430563	0.431888
1.5	0.433193	0.434478	0.435745	0.436992	0.438220	0.439429	0.440620	0.441792	0.442947	0.444083
1.6	0.445201	0.446301	0.447384	0.448449	0.449497	0.450529	0.451543	0.452540	0.453521	0.454486
1.7	0.455435	0.456367	0.457284	0.458185	0.459070	0.459941	0.460796	0.461636	0.462462	0.463273
1.8	0.464070	0.464852	0.465620	0.466375	0.467116	0.467843	0.468557	0.469258	0.469946	0.470621
1.9	0.471283	0.471933	0.472571	0.473197	0.473810	0.474412	0.475002	0.475581	0.476148	0.476705
2.0	0.477250	0.477784	0.478308	0.478822	0.479325	0.479818	0.480301	0.480774	0.481237	0.481691
2.1	0.482136	0.482571	0.482997	0.483414	0.483823	0.484222	0.484614	0.484997	0.485371	0.485738
2.2	0.486097	0.486447	0.486791	0.487126	0.487455	0.487776	0.488089	0.488396	0.488696	0.488989
2.3	0.489276	0.489556	0.489830	0.490097	0.490358	0.490613	0.490863	0.491106	0.491344	0.491576
2.4	0.491802	0.492024	0.492240	0.492451	0.492656	0.492857	0.493053	0.493244	0.493431	0.493613
2.5	0.493790	0.493963	0.494132	0.494297	0.494457	0.494614	0.494766	0.494915	0.495060	0.495201
2.6	0.495339	0.495473	0.495604	0.495731	0.495855	0.495975	0.496093	0.496207	0.496319	0.496427
2.7	0.496533	0.496636	0.496736	0.496833	0.496928	0.497020	0.497110	0.497197	0.497282	0.497365
2.8	0.497445	0.497523	0.497599	0.497673	0.497744	0.497814	0.497882	0.497948	0.498012	0.498074
2.9	0.498134	0.498193	0.498250	0.498305	0.498359	0.498411	0.498462	0.498511	0.498559	0.498605
3.0	0.498650	0.498694	0.498736	0.498777	0.498817	0.498856	0.498893	0.498930	0.498965	0.498999
3.1	0.499032	0.499065	0.499096	0.499126	0.499155	0.499184	0.499211	0.499238	0.499264	0.499289
3.2	0.499313	0.499336	0.499359	0.499381	0.499402	0.499423	0.499443	0.499462	0.499481	0.499499
3.3	0.499517	0.499534	0.499550	0.499566	0.499581	0.499596	0.499610	0.499624	0.499638	0.499651
3.4	0.499663	0.499675	0.499687	0.499698	0.499709	0.499720	0.499730	0.499740	0.499749	0.499758
3.5	0.499767	0.499776	0.499784	0.499792	0.499800	0.499807	0.499815	0.499822	0.499828	0.499835
3.6	0.499841	0.499847	0.499853	0.499858	0.499864	0.499869	0.499874	0.499879	0.499883	0.499888
3.7	0.499892	0.499896	0.499900	0.499904	0.499908	0.499912	0.499915	0.499918	0.499922	0.499925
3.8	0.499928	0.499931	0.499933	0.499936	0.499938	0.499941	0.499943	0.499946	0.499948	0.499950
3.9	0.499952	0.499954	0.499956	0.499958	0.499959	0.499961	0.499963	0.499964	0.499966	0.499967
4.0	0.499968	0.499970	0.499971	0.499972	0.499973	0.499974	0.499975	0.499976	0.499977	0.499978
4.1	0.499979	0.499980	0.499981	0.499982	0.499983	0.499983	0.499984	0.499985	0.499985	0.499986
4.2	0.499987	0.499987	0.499988	0.499988	0.499989	0.499989	0.499990	0.499990	0.499991	0.499991
4.3	0.499991	0.499992	0.499992	0.499993	0.499993	0.499993	0.499993	0.499994	0.499994	0.499994
4.4	0.499995	0.499995	0.499995	0.499995	0.499996	0.499996	0.499996	0.499996	0.499996	0.499996
4.5	0.499997	0.499997	0.499997	0.499997	0.499997	0.499997	0.499997	0.499998	0.499998	0.499998
4.6	0.499998	0.499998	0.499998	0.499998	0.499998	0.499998	0.499998	0.499998	0.499999	0.499999
4.7	0.499999	0.499999	0.499999	0.499999	0.499999	0.499999	0.499999	0.499999	0.499999	0.499999
4.8	0.499999	0.499999	0.499999	0.499999	0.499999	0.499999	0.499999	0.499999	0.499999	0.499999
4.9	0.500000	0.500000	0.500000	0.500000	0.500000	0.500000	0.500000	0.500000	0.500000	0.500000
5.0	0.500000	0.500000	0.500000	0.500000	0.500000	0.500000	0.500000	0.500000	0.500000	0.500000

