MACHAKOS UNIVERSITY

University Examinations for 2020/2021

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

SECOND YEAR SECOND SEMESTER EXAMINATION FOR

BACHELOR OF SCIENCE (ACTUARIAL SCIENCES)

BACHELOR OF SCIENCE (MATHEMATICS)

SMA 230: VECTOR ANALYSIS

DATE: 13/8/2021 **TIME: 8:30 – 10:30 AM**

INSTRUCTION: *Answer Question **ONE** which is compulsory and any other **TWO** Questions*

QUESTION ONE (COMPULSORY) (30 MARKS)

a) Determine the constant a so that the following vector is a solenoid, $V = (-4x - 6y + 3z)i + (-2x + y - 5z)j + (5x + 6y + az)k$ (4 marks)

b) Find p such that the vectors $a = 2i - j + k$; $b = i + 2j - 3k$ and $c = 3i + pj + 5k$ (4 marks)

c) If $\phi = x^2yz - 4xyz^2$. Find the directional derivative of ϕ at $P(1,3,1)$ in the direction of $2i - j - 2k$ (5 marks)

d) Find the area of triangle ABC whose vertices are $A(0,0,0)$, $B(1,1,1)$ and $C(0,2,3)$ (5 marks)

e) Given that $\phi(x, y, z) = xy^2z$ and $A = xi + j + xyk$. Find $\frac{\partial^3}{\partial x^2 \partial z}(\phi A)$ at the point $P(1,2,2)$ (6 marks)

f) Find the work done in moving a particle in the force field given by $F = 3xyi - 5zj + 10xk$ along the curve $x = t^2 + 1$, $y = 2t^2$ and $z = t^3$ from $t = 1$ and $t = 2$ (6 marks)

QUESTION TWO (20 MARKS)

a) Given $R(u) = 3i + (u^3 + 4u^7)j + uk$. Find $\int_1^2 R(u)du$ (4 marks)

b) Verify Green's theorem in the plane for $\oint_C (xy + y^2)dx + x^2dy$ where C is the closed curve of the region bounded by $y = x$ and $y = x^2$ (6 marks)

c) The vector $F = (2xy + z^3)i + x^2 j + 3xz^2 k$

i] Show that it is a conservative force field (3 marks)

ii] find the scalar potential (5 marks)

iii] find the work done in moving an object in this field from $(1, -2, 1)$ to $(3, 1, 4)$ (2 marks)

QUESTION THREE (20 MARKS)

a) If $F = 2zi - xj + yk$, evaluate $\iiint_V F dv$ where v is the region bounded by the surfaces $x = 0, y = 0, x = 2, y = 4, z = x^2, z = 2$ (6 marks)

b) The acceleration of a particle at any time $t \geq 0$ is given by $a = \frac{dr}{dt} = (25 \cos 2t)i + (16 \sin 2t)j + 9tk$. If the velocity v and the displacement r are zero when $t = 0$. Find v and r at any time (6 marks)

c) Find a set of vectors reciprocal to the set of $2i + 3j - k$, $i - j - 2k$, and $-i + 2j + 2k$ (8 marks)

QUESTION FOUR (20 MARKS)

a) A particle is displaced from point $A(2, -3, 1)$ to $B(4, 2, 1)$ under the action of constant forces $12i - 5j + 6k$; $i + 2j - 2k$ and $2i + 8j + k$. Find the work done by the forces on the particle (4 marks)

b) Suppose $F = xyi - zj + x^2 k$ and C is the curve $x = t^2, y = 2t, z = t^3$ from $t = 0$ and $t =$

i) Evaluate the line integrals

$ii)$ $\int_C F \times dr$ (6 marks)

c) Evaluate $\iint_S A.ndS$, where $A = 18zi - 12j + 3yk$ and S is that part of the plane $2x + 3y + 6z = 12$ which is located in the first octant (10 marks)

QUESTIONS FIVE (20 MARKS)

a) Find the moment about the point $A(5, -1, 3)$ of a force $4i + 2j + k$ through point $B(5, 2, 4)$ (3 marks)

b) Use Divergence theorem to evaluate $\iint_S F \cdot dS$ where $F = 4xi - 2y^2 j + z^2 k$ and S is the surface bounding the region $x^2 + y^2 = 4$, $z = 0$and $z = 3$ (8 marks)

c) Use Stokes's theorem to evaluate$\int_C v \cdot dr$ where $v = y^2 i + xyj + xzk$ and C is the bounding curve of the hemisphere $x^2 + y^2 + z^2 = 0$ oriented in the positive direction (9 marks)