MACHAKOS UNIVERSITY

ISO 9001:2008 Certified

University Supplementary Examinations

SMA 203 LINEAR ALGEBRA II

Department of Mathematics and Statistics.

Time 2hours

Attempt question ***one (compulsory)*** and any other ***two questions***.

Question 1(30marks)

a.) State the Cayley Hamilton theorem (3marks)

b.) Let $A=\begin{pmatrix}1 & 2\\ 3 & 4\end{pmatrix}$ and let $g(x)=x^2-5x-2, \quad f(x)=2x^2-3x+7$. Show that A is a root of $g(x)$ and not a root of $f(x)$ (4marks)

c.) Express $(2,-5,3)$ as a linear combination of $(2,-3,2)(2,-4,1)$ and $(1,-5,7)$ (4marks)

d.) Let $f:V\to V$ be a linear mapping. Let A be the matrix relative to a basis $v_1\ v_2\ v_3\ .....v_n$. Prove that the columns of A span $im\ f$ (4marks)

e.) Define a linear mapping (3marks)

f.) Let $v=\Re^4$; $w=\{(x,\ y,\ z,\ w)/x+y-z=0\}$ by $f(x,y)=(x+y,0)$. show that w is a subspace of $\Re^4$ (5marks)

g.) Prove that "any set of linearly independent vectors in V can be extended into a basis for V" (4marks)

h.) Let $V=P_3(x)$ and define $D:v\to v$ by $Df=\dfrac{df}{dx}$, D is linear. Determine

i) Nullity (2marks)

ii) Rank (1mark)

Question 2(20mks)

a.) Define $f:\Re^3 \to \Re^3$ by $f(x,y,z)=(x+y,z,-x)$. Let $S=\{(1,1,1),(1,2,1),(0,-1,1)\}$ be an ordered basis for $\Re^3$. Determine the matrix of f relative to the basis S. (8marks)

b.) If $f: v \to w \quad (v,w \textit{ vector spaces})$ prove that;

i. $\ker f$ is a subspace of v (6marks)

ii. $im\ f$ is a subspace of w (6marks)

Question 3(20marks)

a.) Determine the minimal polynomial of the matrix

$$A=\begin{pmatrix} 2 & 1 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & -2 & 4 \end{pmatrix}$$ (7marks)

b.) Let V be a subspace of R^4 spanned by $(1,-1,0,0)$ $(0,0,1,-1)$ $(0,0,3,1)$ and let u be the subspace of R^4 spanned by $(1,2,-2,3)$ $(3,0,2,-1)$ $(2,1,0,1)$. Determine a basis for;

i) $U+V$ (6marks)

ii) $U\ n\ V$ (7marks)

Question 4 (20marks)

a.) Consider the matrix $A=\begin{pmatrix} -1 & -5 \\ 10 & 14 \end{pmatrix}$. Work out the matrices P *and* P^{-1} such that $P\ A\ P^{-1}=D$. Where D is a diagonal matrix. (6marks)

b.) Prove the dimension's formula $Dim(u+v)=\dim u+\dim v-\dim(u\,n\,v)$ (7marks)

c.) Calculate the eigen values and the eigen vectors of the matrix

$$A=\begin{pmatrix} 1 & 3 \\ 4 & 2 \end{pmatrix}$$ (7marks)

Question 5(20marks)

a.) Let $f:\Re^n \to \Re^m$ have the matrix $A=\begin{pmatrix} 2 & 1 & -1 & 3 \\ 3 & 0 & 1 & 4 \\ 1 & 2 & -3 & 2 \end{pmatrix}$ with respect to standard basis.

Determine;

i) The value of m and n (3marks)

ii) The $im\, f$ and $rank\ f$ (5marks)

iii) The $\ker f$ and $nullity\ f$ (5marks)

b.) Let V be a vector space and let w be a fixed vector in V. Define

$W=\{set\ of\ all\, vectors\ in\ V\ which\ are \perp W\}$

$v_1 \perp v_2 \iff v_1\ is\ perpendicular\ to\ v_2$.

$w=\{v\in v\ /\ v\cdot w=0\}$. Show that w is a subspace of v. (7marks)