MACHAKOS UNIVERSITY

University Examinations 2017/2018

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

THIRD YEAR SECOND SEMESTER EXAMINATIONS FOR

BACHELOR OF SCIENCE INMATHEMATICS AND COMPUTER SCIENCE

BACHELOR OF SCIENCE IN MATHEMATICS

BACHELOR OF EDUCATION

BACHELOR OF ARTS

SMA 305: COMPLEX ANALAYSIS I

DATE: 4/12/2017 **TIME: 8.30-10.30 AM**

INSTRUCTIONS: Answer QUESTION ONE and any other TWO QUESTIONS

QUESTION ONE 30 MARKS (COMPULSORY)

a) Factorize $16x^2 + 25y^2$ (2 marks)

b) Simplify $\dfrac{5+3i}{-i}$ (2 marks)

c) For what values of z if $f(z) = \dfrac{3z^2+1}{z^2+1}$ is continuous (3 marks)

d) Explain the nature of the transformation $w = z^2$ (5 marks)

e) Simplify $\dfrac{(\cos 4\theta + i\sin 4\theta)(\cos 3\phi + i\sin 3\phi)}{(\cos\theta + i\sin\theta)(\cos 2\phi + i\sin 2\phi)}$ (5 marks)

f) Let $f(z) = u + iv$ be an analytic function. If $u = x^3 - 3x^2 y,$ find $f(z)$ (5 marks)

g) Find the cube roots of -1 (8 marks)

QUESTION TWO (20 MARKS)

a) Consider the function $f(z)=\frac{z-\sin z}{z^2}$

i. State the singularity of $f(z)$ (2 marks)

ii. What kind of the singularity of $f(z)$ (2 marks)

iii. Find the Laurent series of $f(z)$ (3 marks)

iv. What is the region of convergence of $f(z)$ (3 marks)

b) Find the fixed points of the bilinear transformation $w=\frac{3z-4}{z-1}$ (5 marks)

c) Prove that $f(z)=2z^2$ is continuous at $z=z_o$ (5 marks)

QUESTION THREE (20 MARKS)

a) Find the image of the point $(4,3)$ on $z-$plane under the transformation $w=2z^2+3$

(4 marks)

b) Expand $f(z)=\frac{3}{z^2(z-3)^2}$ in a Laurent series at $z=3$ (6 marks)

c) Evaluate $\int \bar{z}dz$ from $z=0$ to $z=4+2i$ along the line $z=2i$ to $z=4+2i$ (10 marks)

QUESTION FOUR (20 MARKS)

a) Express $z=4+3i$ in polar form (5 marks)

b) Show that $f(z)=e^z$ is a periodic function with period $2k\pi i$ (5 marks)

c) Find the bilinear transformation which maps the points $z_1=2, z_2=i, z_3=-2$ into the points $w_1=1, w_2=i, w_3=-1$ respectively. (10 marks)

QUESTION FIVE (20 MARKS)

a) Prove that $\sin z=\sin x\cosh y-i\cos x\sinh y$ (6 marks)

b) Let $f(z)=\frac{z^2-7z+10}{(z-3)^2}$

Determine the poles of $f(z)$. calculate the pole of $f(z)$ at it poles. (6 marks)

c) Prove that $\int_0^\infty \frac{dx}{x^4+1}=\frac{\pi}{\sqrt{2}}$ (8 marks)