MACHAKOS UNIVERSITY

University Examinations for 2016/2017 Academic Year

SCHOOL OF BUSINESS AND ECONOMICS

DEPARMENT OF ECONOMICS

THIRD YEAR FIRST SEMESTER EXAMINATIONS FOR

BACHELOR OF ECONOMICS & STATISTICS

BACHELOR OF ECONOMICS & FINANCE

BACHELOR OF ECONOMICS

EES 300: MATHEMATICS FOR ECONOMISTS III

DATE: 27/7/2017 **TIME: 11:00 – 1:00 PM**

INSTRUCTIONS:

Answer Question ONE and any other TWO questions

QUESTION ONE (COMPULSORY) (30 MARKS)

a) The marginal costs of two firms are given by the following functions:

1) $C'(Q) = 15 + 20Q - 12Q^2$ $TC = 100$ when $Q = 0$

2). $C'(Q) = 100e^{0.2Q}$ $TC = 3000$ when $Q = 0$

i) Find the total cost function C(Q) for each firm. (4 marks)

ii) Find the average cost function for each firm (4 marks)

iii) What is the fixed costs of each firm (2 marks)

b) Find the time path of capital K(t) given the following rates of net investment flow functions

i) $\mathrm{I(t)} = 5\mathrm{t}^{1/2}$ $\mathrm{K}(0) = 500$ (3 marks)

ii) $\mathrm{I(t)} = 18\mathrm{t}^{3/5}$ $\mathrm{K}(0) = 30$ (3 marks)

iii) For each of (i) to (ii) above, find the amount of capital formation over the interval [0,5] (6 marks)

c) Find the producer surplus of the following

$Q = \sqrt{P - 6}$ given that $\overline{P} = 20$ (4 marks)

d) Verify that the following differential equation is exact and solve the equations

$8ytdy + (4y^2 + 3t)dt = 0$ (4 marks)

QUESTION TWO (20 MARKS)

a) Derive general solution of the First order differential equations (FOLDE) (6 marks)

b) Suppose you are given the following demand and supply functions

$Qd = \alpha - \beta P \qquad (\alpha, \beta > 0)$

$Qs = -\gamma + \delta P \qquad (\gamma, \delta > 0)$

i) Assuming that the rate of change of price over time is directly proportional to the excess demand, find the time path P(t) (general solution (6 marks)

ii) What is the inter-temporal equilibrium price (2 marks)

iii) What is the market clearing equilibrium price (2 marks)

iv) Does the market have a dynamically stable equilibrium price? Explain (4 marks)

QUESTION THREE (20 MARKS)

a) Find the general and definite solution to the following differential equations

i) $\frac{dy}{dt} + 4y = 10 \qquad y(0) = 12$ (2 marks)

ii) $\frac{dy}{dt} + 10y = 12 \qquad y(0) = 20$ (2 marks)

iii) $\frac{dy}{dt} = 5y - 10 \qquad y(0) = 3$ (2 marks)

b) Find the integral of the following

i) $\int \left(8x^2 e^{(X^3+10)} + \frac{4}{x^3}\right) dx \qquad (x \neq 0)$ (2 marks)

ii) $\int 5e^{10x+3}\, dx$ (2 marks)

iii) $\int x(x^2+3)^{\frac{1}{2}} dx$ (2 marks)

iv) $\int_2^4 x\, 1n\, x \; dx$ (2 marks)

c) Solve the following equations for x (6 marks)

i. $2e^{2x-100} = 300$

ii. $z = ae^{bx}$

QUESTION FOUR (20 MARKS)

a) Solve the following difference equations using iterations.

i) $y_{t+1} = y_t + 4 \qquad (y_0 = 8)$ (2 marks)

ii) $y_t = 3y_{t-1} + 4 \qquad (y_0 = 1)$ (2 marks)

iii) $y_{t+1} = 0.5y_t + 12 \qquad (y_0 = 9)$ (2 marks)

iv) $y_{t+1} = 3y_t + 4 \qquad (y_0 = 10)$ (2 marks)

b) For the general first difference equation given as follows:

$$y_{t+1} + \alpha y_t = \beta$$

i) Find the general solution in the case where $(a \neq -1)$ (6 marks)

ii) Decompose the general solution into two components, the complementary function y_c and the particular integral y_p and interpret each term. (4 marks)

iii) Which of the two components in (ii) above determines whether the equilibrium is dynamically stable or not (2 marks)

QUESTION FIVE (20 MARKS)

(a) Owing to the change in taste and attitude towards wine, the value of wine belonging to a major wine dealer in some small religious province decline according to the following function $V_t = V_0e^{-rt}$, where V_t is value of wine at time t, V_0 is value of wine at time 0, r is rate of decline in percentage terms and t is time.

i) After how long will the value of wine be 80 percent of the initial value given a rate of decline of r=0.65 percent. (5 marks)

ii) The wine dealer decides that as soon as the value of his wine is 70 percent the initial value, he will close his business. After how long will he close his business if the rate of decline is 0.65 percent? (5 marks)

(b) Given the following IS-LM model;

Goods Market	**Money market**
$Y = C + I$	$Md = 100 + 0.15Y - 0.25r$
$C = 200 + 0.2Y$	$Ms = 120$
$I = 8 - 0.3r$	$Md = Ms$

Find Y and r by Cramer's rule and matrix inverse methods. (10 marks)