



MACHAKOS UNIVERSITY

University Examinations 2021/2022 Academic Year

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

THIRD YEAR FIRST SEMESTER EXAMINATION FOR

BACHELOR OF SCIENCE (ACTUARIAL SCIENCE)

SAC 400: MATHEMATICS FOR DEMOGRAPHY AND GRADUATION

DATE: 26/8/2022

TIME: 8.30-10.30 AM

INSTRUCTION:

Answer **ALL** Questions from Section A and any other **TWO** Questions in Section B.

All questions in Section B carry equal marks.

Duration of the examination: 3 hours

SECTION A (30 Marks). Answer ALL questions.

QUESTION ONE (30 Marks)

- a) Define the following terms (3 marks)
 - i. Mortality
 - ii. Cohort
 - iii. Life table
- b) Distinguish between fertility and fecundity (2 marks)
- c) Give the merit and demerit of using Crude Birth Rate (CBR) as a measure of fertility? (2 marks)
- d) Explain the meaning of Crude Birth Rate (CBR) when it is given as 21% (3 marks)
- e) Identify and describe briefly three sources of demographic data (6 marks)
- f) Give one advantage of using population register (1 mark)
- g) State any three uses of vital statistics (3 marks)
- h) Distinguish between complete and bridged life tables. (2 marks)
- i) Identify and describe briefly two types of enumeration (4 marks)

- j) Suppose in a baseline enumeration, the total population at the beginning of the survey in 01/01/2009 was 9993. After twelve months, the population was counted again it was found to be 11673. Estimate the mid-year population. (3 marks)
- k) In a certain developing country, census was taken in 1999 and 2009, the ratio $\frac{p_{10}}{p_0} = 1.7573$. Compute the annual growth rate γ . (3 marks)

SECTION B (40 Marks) Answer any TWO questions)

QUESTION TWO (20 MARKS)

- a) Distinguish between Gross Reproductive Rate and Total Fertility Rate (4 marks)
- b) Consider the following table on data from a West African country, assume that the total population is 7000

Age group	Number of women	Number of births	ASFR	Number of female births	Female ASFR	$\frac{l_x}{l_0}$
15-19	250	30	0.10	10	0.045	0.90
20-24	300	70	0.27	35	0.117	0.87
25-29	350	90	0.33	45	0.150	0.85
30-34	400	80	0.20	35	0.088	0.83
35-39	500	40	0.05	11	0.023	0.80
Total	1800	310	0.95	136	0.423	

Calculate;

- Crude birth rate (3 marks)
- General fertility rate (3 marks)
- Gross reproductive rate (3 marks)
- Total fertility rate (3 marks)
- Net reproductive rate (4 marks)

QUESTION THREE (20 MARKS)

- a) Give the uses of life tables (2 marks)
- b) Define clearly each of the following life table functions (6 marks)
- l_x
 - ${}_n d_x$
 - ${}_n q_x$
 - ${}_n T_x$
 - ${}_n L_x$
 - e_x^0
- c) What is the disadvantage of exponential model (1 mark)
- d) Assume that in 1990 and 2009, the world population was 3.9067 and 7.8258 billion respectively.
- Obtain the exponential model (7 marks)
 - Forecast the future population in 2040 (2 marks)
 - Determine the year in which the population would be 12 billion (2 marks)

QUESTION FOUR (20 MARKS)

- a) Consider the life table below and fill in the blanks marked with question marks. (4 marks)

Age x	l_x	d_x	p_x	q_x	L_x	T_x	e_x
40	95000	500	?	?	?	4850300	?
41	?	400	?	?	?	?	?

- b) Given the following life table on Australian males

Age x	l_x	d_x	p_x	q_x
30	96726	159	0.99835	0.00165
31	96567	155	0.99839	0.00161
32	96412	161	0.99833	0.00167
33	96251	168	0.99825	0.00175
34	96083	173	0.99820	0.00180
35	95910	182	0.99810	0.00190

Calculate

- i. Probability of males aged 32 who will survive to age 35 (4 marks)
- ii. Probability of males aged 32 who will die by age 35 (4 marks)
- iii. Out 4000 males aged 31, how many will die after age 35? (4 marks)
- iv. Probability of males aged 32 who will die by age 34 (4 marks)

QUESTION FIVE (20 MARKS)

- a) Define parity, zero parity and also give an expression for parity progression ratio (PPR) (6 marks)
- b) Given the parity and number of women with at least i children in France in 1993

Number of women having	Parity x	Number of women with x children	Number of women with at least i children	PPR $\frac{Y}{X}$	Number of women with exactly x children
$\geq 0 = 1000$	0	192509	$1101680 = Y$	0.824	176
$\geq 1 = 824$	1	279338	$907785 = X$	0.690	255
$\geq 2 = 569$	2	255318	626372	0.590	233
$\geq 3 = 336$	3	157082	369559	0.588	138
$\geq 4 = 198$	4	85245	217301	0.605	78
$\geq 5 = 120$	5	48167	131467	0.628	45

- i) Compute the average family size (6 marks)
- ii) Calculate the PPR and comments on your results (8 marks)